

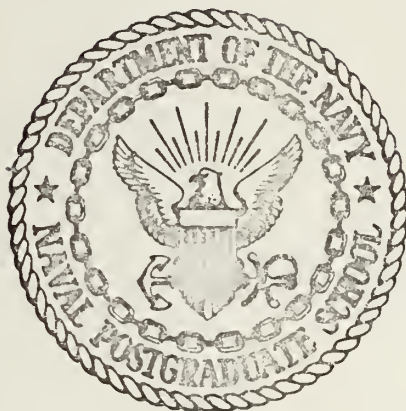
HYBRID MODE ANALYSIS OF MICROSTRIP ON  
DIELECTRIC AND FERRITE SUBSTRATE

Ahmet M nir T fek io lu

Postgraduate School  
California 93940

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

HYBRID MODE ANALYSIS OF MICROSTRIP  
ON  
DIELECTRIC AND FERRITE SUBSTRATE

Ahmet Münir Tüfekçioğlu

September 1974

Thesis Advisor:

J. B. Knorr

Approved for public release; distribution unlimited.

T164059



UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Hybrid Mode Analysis of Microstrip on Dielectric and Ferrite Substrate		5. TYPE OF REPORT & PERIOD COVERED Electrical Engineer; September 1974
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Ahmet Münir Tüfekcioğlu		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE September 1974
		13. NUMBER OF PAGES 95
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)  Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)  Microstrip Spectral Domain Transform Method of Moments		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number)  A hybrid mode analysis of microstrip on a dielectric substrate is presented. A numerical solution for wavelength and characteristic impedance of single and coupled, balanced strips is obtained. Line parameters are shown to be very frequency dependent but in agreement with the quasi-static results of other investigators in the low frequency limit.		



Block #20 Continued.

The extension of this technique to perturbation analysis of microstrip on ferrite is also described. A numerical solution for the propagation constant is obtained.



Hybrid Mode Analysis of Microstrip  
on  
Dielectric and Ferrite Substrate

by

Ahmet Münir Tüfekçioğlu  
Lieutenant (jg), Turkish Navy  
B.S.E.E., Naval Postgraduate School, 1973  
M.S.E.E., Naval Postgraduate School, 1973

Submitted in partial fulfillment of the  
requirements for the degree of

ELECTRICAL ENGINEER

from the

NAVAL POSTGRADUATE SCHOOL  
September 1974

Thesis  
185  
c-1

## ABSTRACT

A hybrid mode analysis of microstrip on a dielectric substrate is presented. A numerical solution for wavelength and characteristic impedance of single and coupled, balanced strips is obtained. Line parameters are shown to be very frequency dependent but in agreement with the quasi-static results of other investigators in the low frequency limit.

The extension of this technique to perturbation analysis of microstrip on ferrite is also described. A numerical solution for the propagation constant is obtained.



## TABLE OF CONTENTS

I.	INTRODUCTION.....	10
II.	DISPERSION CHARACTERISTICS ON DIELECTRIC SUBSTRATE.....	14
	A. FIELD AND BOUNDARY CONDITIONS.....	14
	B. SPECTRAL DOMAIN TRANSFORM.....	17
	C. PHYSICAL PARAMETERS.....	25
	D. THE CHARACTERISTIC IMPEDANCE IN TERMS OF DISPERSION CHARACTERISTICS.....	33
III.	PERTURBATION ANALYSIS OF MICROSTRIP ON FERRITE SUBSTRATE.....	45
IV.	COMPUTER PROGRAMMING.....	52
V.	NUMERICAL RESULTS AND COMPARISON.....	56
VI.	CONCLUSIONS.....	75
	APPENDIX A: AUXILIARY VECTOR POTENTIAL FUNCTIONS.....	76
	APPENDIX B: TRANSVERSE ELECTRIC AND MAGNETIC FIELDS...	80
	APPENDIX C: AVERAGE POWER EXPRESSION IN REGION 2 (TRIGONOMETRIC CASE).....	84
	APPENDIX D: COMPUTER PROGRAM.....	87
	LIST OF REFERENCES.....	94
	INITIAL DISTRIBUTION LIST.....	95



## LIST OF TABLES

### TABLE

I.	Characteristic Impedance of Microstrip Transmission Lines Single Strip.....	68
II.	Effective Dielectric Constants and Characteristic Impedances for Coupled Strips.....	69
III.	Frequency Dependence of Single and Coupled Strips.....	70
IV.	Sample Computer Output for Single Strip.....	71
V.	Sample Computer Output for Coupled Strips (Odd Mode).....	72
VI.	Sample Computer Output for Coupled Strips (Even Mode).....	73



## LIST OF FIGURES

### Figure

1.	Three Dimensional Microstrip Transmission Line..	11
2.	Coupled Microstrip and Regions.....	12
3.	Single and Coupled Strips Even and Odd Transverse Current Distributions.....	28
4.	$G_4(\alpha, \lambda/\lambda')$ versus $\alpha$ .....	31
5.	$G_4(\alpha, \lambda/\lambda')  J_z(\alpha) ^2$ versus $\alpha$ .....	32
6.	Region 1 Average Power Distribution in $\alpha$ -Domain.	40
7.	Region 2 Average Power Distribution in $\alpha$ -Domain.	41
8.	$P_{1_{AVE}}$ versus $\alpha D$ .....	43
9.	$P_{2_{AVE}}$ versus $\alpha D$ .....	44
10.	Loss Per Wavelength in db versus Frequency (GHz).....	50
11.	$\beta_f/\beta$ versus Frequency (GHz).....	51
12.	Single Strip $\lambda'/\lambda$ versus $D/\lambda$ curves $\epsilon_{r_2} = 6.0$ ...	57
13.	Single Strip $Z_o$ versus $D/\lambda$ curves $\epsilon_{r_2} = 6.0$ ....	58
14.	Single Strip $\lambda'/\lambda$ versus $D/\lambda$ curves $\epsilon_{r_2} = 12.0$ ..	59
15.	Single Strip $Z_o$ versus $D/\lambda$ curves $\epsilon_{r_2} = 12.0$ ....	60
16.	Coupled Strip Even and Odd Modes $\lambda'/\lambda$ versus $D/\lambda$ curves $\epsilon_{r_2} = 6.0$ $W/D = 1.54$ .....	61
17.	Coupled Strip Even and Odd Modes $Z_o$ versus $D/\lambda$ Curves $\epsilon_{r_2} = 6.0$ $W/D = 1.54$ .....	62
18.	Coupled Strip Even and Odd Modes $\lambda'/\lambda$ versus $D/\lambda$ curves $\epsilon_{r_2} = 12.0$ $W/D = 0.8$ .....	63
19.	Coupled Strip Even and Odd Modes $Z_o$ versus $D/\lambda$ curves $\epsilon_{r_2} = 12.0$ $W/D = 0.8$ .....	64
20.	Three Dimensional Plot $\lambda'/\lambda$ , $W/D$ , $D/\lambda$ Single Strip.....	65



Figure

21. Three Dimensional Plot,  $Z_o$ ,  $W/D$ ,  $D/\lambda$  Single  
Strip..... 66



## ACKNOWLEDGEMENT

The author gratefully expresses his gratitude to Professor Jeffrey B. Knorr for his considerable help and guidance during this study and his patience in editing this thesis.

Also of great help has been the encouragement in this work of my wife, Adalet, who carefully and patiently typed the first draft.



## I. INTRODUCTION

The analysis of microstrip is of great importance as this type of transmission line has found wide use due to its compatibility with microwave integrated circuitry. The open microstrip transmission line has been predominant because it can be etched or deposited easily on the substrate.

The early analytic work on microstrip lines has been based on a proposed TEM mode of propagation which is, in essence, a static approximation to a dynamic system. For sufficiently low frequencies the quasi-static theory can be employed to obtain the characteristics of microstrip lines. When the wavelength in a microstrip line becomes comparable to transverse dimensions of the line the deviation from quasi-static behavior becomes significant and high order modes of propagation become possible.

In this present work a Spectral Domain transform method is applied for calculating the frequency dependent characteristics of single and coupled microstrip transmission lines. Effects of geometry on the dispersion and characteristic impedance have been analyzed.

Figures 1 and 2 show the geometry of the microstrip line. The strip conductor is assumed to be infinitely thin and perfectly conducting and the ground plane and dielectric or ferrite substrate are assumed to be infinite in extent.



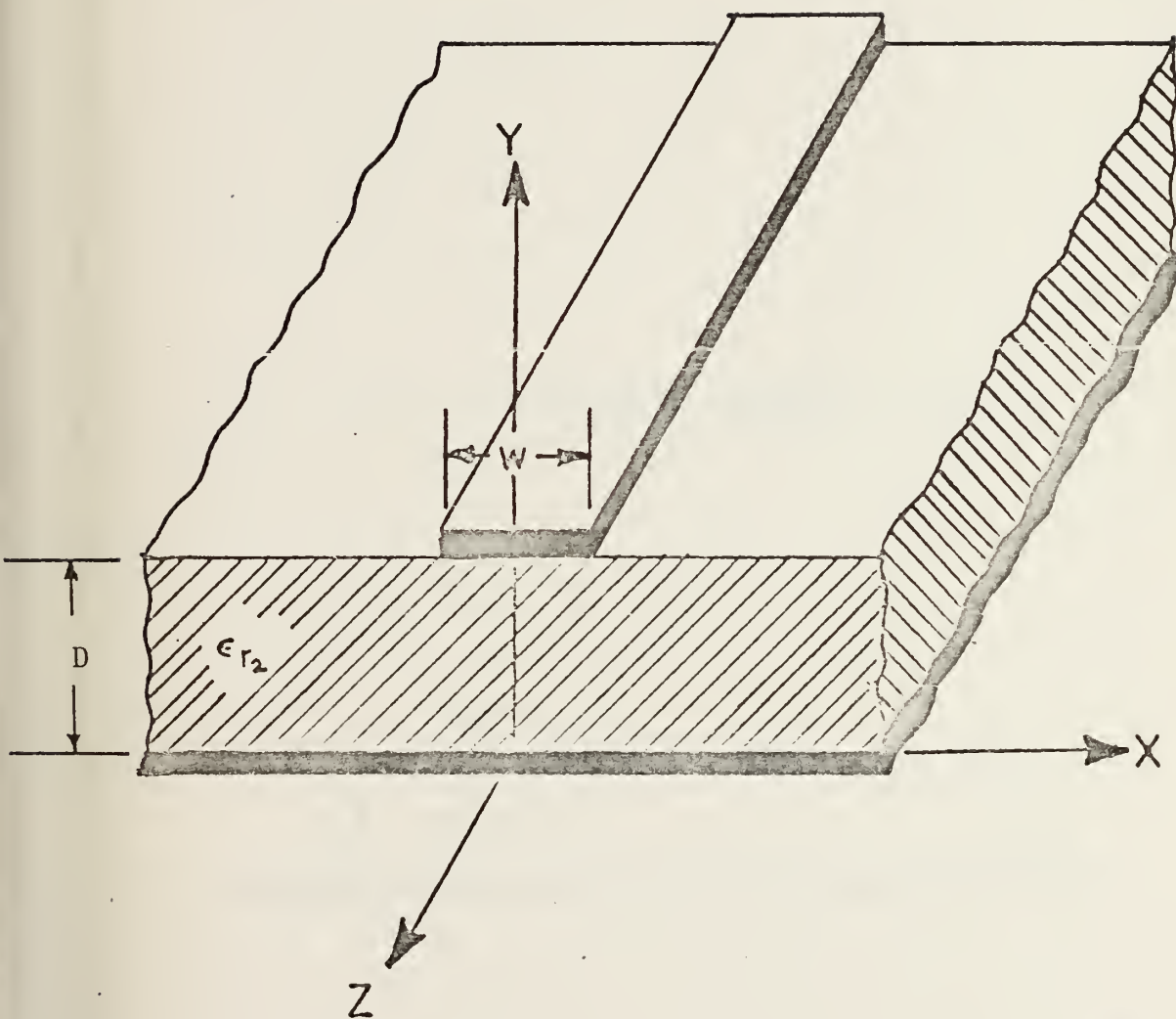


Figure 1. Three Dimensional Microstrip Transmission Line.



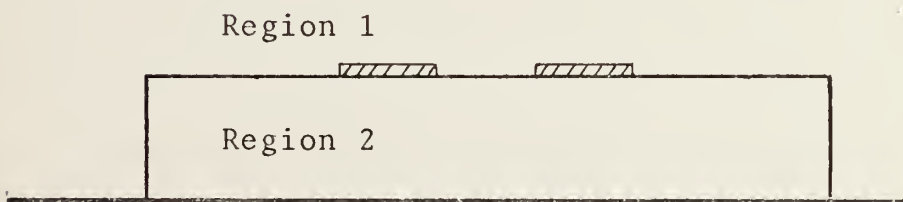
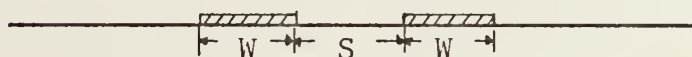
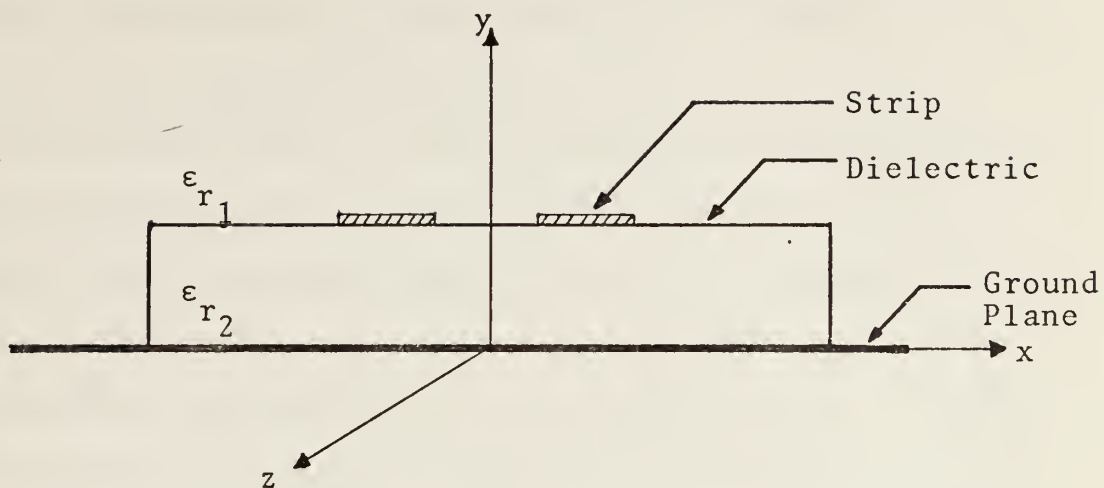


Figure 2. Coupled Microstrip and Regions.



It is assumed initially that the substrate material is lossless and isotropic. Lossy anisotropic substrates are treated later using perturbation theory.

The Spectral Domain transform method was suggested by Itoh and Mittra, [Ref. 6] and yields a solution to the Boundary value problem of microstrip via the method of moments. The method was used by Itoh and Mittra to find the dispersion characteristics of a single strip. In this work the method has been extended to cover dispersion and characteristic impedance of both single and coupled microstrips. The case of a microstrip on a ferrite substrate has also been solved using perturbation theory.



## II. DISPERSION CHARACTERISTICS ON DIELECTRIC SUBSTRATE

### A. FIELD AND BOUNDARY CONDITIONS

Let the following electric and magnetic fields exist and propagate in the z-direction as justified by the Hertzian Vector Potential functions described in Appendix A.

$$E_z = k_c^2 \phi^e e^{\gamma z} \quad (1)$$

$$H_z = k_c^2 \phi^e e^{\gamma z} \quad (2)$$

where  $\gamma$  is the Propagation constant. For the lossless substrate material

$$\gamma = j\beta.$$

From the field expressions in equation (1) and (2) all other components of electric and magnetic field can be derived from Maxwell's curl equations, as shown in Appendix B and leads to the following equations.

$$E_x = \left( \gamma \frac{\partial \phi^e}{\partial x} - j\omega\mu \frac{\partial \phi^h}{\partial y} \right) e^{\gamma z} \quad (3)$$

$$H_x = \left( \gamma \frac{\partial \phi^h}{\partial x} + j\omega\epsilon \frac{\partial \phi^e}{\partial y} \right) e^{\gamma z} \quad (4)$$

$$E_y = \left( \gamma \frac{\partial \phi^e}{\partial y} + j\omega\mu \frac{\partial \phi^h}{\partial x} \right) e^{\gamma z} \quad (5)$$

$$H_y = \left( \gamma \frac{\partial \phi^h}{\partial y} - j\omega\epsilon \frac{\partial \phi^e}{\partial x} \right) e^{\gamma z} . \quad (6)$$



Applying boundary conditions at the interface between region 2 and the ground plane, tangential electric fields must be zero, so it follows that at  $y = 0$ ,

$$E_{z_2}(x, 0, z) = 0 \quad (7)$$

$$E_{x_2}(x, 0, z) = 0. \quad (8)$$

Also at the interface between regions 1 and 2, tangential electric fields must be continuous:

At  $y = D$ ,

$$E_{z_1}(x, D, z) = E_{z_2}(x, D, z) \quad (9)$$

$$E_{x_1}(x, D, z) = E_{x_2}(x, D, z). \quad (10)$$

Also the electric fields will exist only in the dielectric part of the interface and can be expressed as,

$$E_{z_1}(x, D, z) = \begin{cases} 0 & \text{on strip} \\ e_z(x)e^{\gamma z} & \text{elsewhere} \end{cases} \quad (11)$$

$$E_{x_1}(x, D, z) = \begin{cases} 0 & \text{on strip} \\ e_x(x)e^{\gamma z} & \text{elsewhere} \end{cases} \quad (12)$$

Similarly, tangential magnetic fields must be discontinuous by corresponding surface current densities

$$H_{z_1}(x, D, z) - H_{z_2}(x, D, z) = \begin{cases} J_x(x)e^{\gamma z} & \text{on strip} \\ 0 & \text{elsewhere.} \end{cases} \quad (13)$$



$$H_{x_1}(x,D,z) - H_{x_2}(x,D,z) = \begin{cases} J_z(x) e^{\gamma z} & \text{on strip} \\ 0 & \text{elsewhere.} \end{cases} \quad (14)$$

Substituting the field expressions of equation (1) through (6) into the boundary condition expressions of equations (7) through (14), one obtains

$$k_{c_2}^2 \phi_2^e(x,0) = 0 \quad (15)$$

$$\gamma \frac{\partial \phi_2^e}{\partial x}(x,0) - j\omega\mu_2 \frac{\partial \phi_2^h}{\partial y}(x,0) = 0 \quad (16)$$

$$k_{c_1}^2 \phi_1^e(x,D) = k_{c_2}^2 \phi_2^e(x,D) \quad (17)$$

$$\gamma \frac{\partial \phi_1^e(x,D)}{\partial x} - j\omega\mu_1 \frac{\partial \phi_1^h(x,D)}{\partial y} = \gamma \frac{\partial \phi_2^e(x,D)}{\partial x} - j\omega\mu_2 \frac{\partial \phi_2^h(x,D)}{\partial y} \quad (18)$$

$$k_{c_1}^2 \phi_1^e(x,D) = \begin{cases} 0 & \text{on strip} \\ e_z(x) & \text{elsewhere} \end{cases} \quad (19)$$

$$\gamma \frac{\partial \phi_1^e(x,D)}{\partial x} - j\omega\mu_1 \frac{\partial \phi_1^h(x,D)}{\partial y} = \begin{cases} 0 & \text{on strip} \\ e_x(x) & \text{elsewhere} \end{cases} \quad (20)$$

$$k_{c_1}^2 \phi_1^h(x,D) - k_{c_2}^2 \phi_2^h(x,D) = \begin{cases} J_x(x) & \text{on strip} \\ 0 & \text{elsewhere} \end{cases} \quad (21)$$



$$\begin{aligned} & \left( \gamma \frac{\partial \phi_1^h(x,D)}{\partial x} + j\omega\epsilon_1 \frac{\partial \phi_1^e(x,D)}{\partial y} \right) \\ & - \left( \gamma \frac{\partial \phi_2^h(x,D)}{\partial x} + j\omega\epsilon_2 \frac{\partial \phi_2^e(x,D)}{\partial y} \right) = \begin{cases} J_z(x) & \text{on strip} \\ 0 & \text{elsewhere} \end{cases} \quad (22) \end{aligned}$$

## B. SPECTRAL DOMAIN TRANSFORM

All potential functions must satisfy the following relation:

$$\nabla_t^2 \phi + k_c^2 \phi = 0 \quad (23)$$

where

$$k_c^2 = \gamma^2 + k^2 = k^2 - \beta^2. \quad (24)$$

One introduces the Fourier transform to the  $\alpha$ -domain, as suggested by Itoh and Mittra:

$$\phi_i(\alpha, y) = \int_{-\infty}^{+\infty} \phi_i(x, y) e^{j\alpha x} dx \quad i = 1, 2, 3, \dots \quad (25)$$

Thus, the transform of equation (23) becomes

$$F_x \left[ \frac{\partial^2 \phi}{\partial x^2} \right] + F_x \left[ \frac{\partial^2 \phi}{\partial y^2} \right] + k_c^2 F_x [\phi] = 0 \quad (26)$$

$$\begin{aligned} & (j\alpha)^2 F_x [\phi(x, y)] + \frac{\partial^2}{\partial y^2} F_x [\phi(x, y)] \\ & + k_c^2 F_x [\phi] = 0 \end{aligned} \quad (27)$$

where

$$F_x \left[ \frac{\partial \phi(x, y)}{\partial x} \right] = -j\alpha F_x [\phi(x, y)] \quad (28)$$

has been applied.



Clearly, one can obtain  $\alpha$ -domain representation of equation (23),

$$-\alpha^2 \Phi(\alpha, y) + \frac{\partial^2}{\partial y^2} \Phi(\alpha, y) + k_C^2 \Phi(\alpha, y) = 0 \quad (29)$$

or

$$\frac{\partial^2 \Phi(\alpha, y)}{\partial y^2} = (\alpha^2 - k_C^2) \Phi(\alpha, y). \quad (30)$$

Equation (30) should be analyzed carefully both for Region 1 and Region 2.

For Region 1, (free space propagation constant),

$$\gamma_1^2 = \alpha_1^2 - k_{C1}^2 = \alpha^2 + \beta^2 - k_1^2 \quad (31)$$

where

$$k_1 = \omega \sqrt{\mu_0 \epsilon_0} \quad \text{and} \quad \beta = \frac{2\pi}{\lambda'}$$

$$\gamma_1^2 = \alpha^2 + \left( \frac{2\pi}{\lambda} \right)^2 \left[ \left( \frac{\lambda}{\lambda'} \right)^2 - 1 \right]. \quad (32)$$

$\lambda'$  is the microstrip wavelength and is related to the free-space wavelength  $\lambda$  as

$$\lambda \geq \lambda' \geq \frac{\lambda}{\sqrt{\epsilon_r}}. \quad (33)$$

By substituting Equation (33) into (32) one can find lower and upper values for  $\gamma_1$

$$\alpha^2 + (\epsilon_{r1} - 1) \left( \frac{2\pi}{\lambda} \right)^2 \geq \gamma_1^2 \geq \alpha^2. \quad (34)$$

Therefore  $\gamma_1$  is always a real quantity independent of the values of  $\alpha$ .



Similarly for Region 2,

$$\gamma_2^2 = \alpha^2 - \left( \frac{2\pi}{\lambda} \right)^2 \left[ \epsilon_{r_2} - \left( \frac{\lambda}{\lambda_1} \right)^2 \right] \quad (35)$$

so it is clear that  $\gamma_2$  will be imaginary for

$$-\frac{2\pi}{\lambda} \sqrt{\epsilon_{r_2} - \left( \frac{\lambda}{\lambda_1} \right)^2} < \alpha < \frac{2\pi}{\lambda} \sqrt{\epsilon_{r_2} - \left( \frac{\lambda}{\lambda_1} \right)^2} \quad (36)$$

and will be real when  $\alpha$  is in the range

$$-\infty < \alpha < -\frac{2\pi}{\lambda} \sqrt{\epsilon_{r_2} - \left( \frac{\lambda}{\lambda_1} \right)^2} ; \frac{2\pi}{\lambda} \sqrt{\epsilon_{r_2} - \left( \frac{\lambda}{\lambda_1} \right)^2} < \alpha < +\infty. \quad (37)$$

For Region 1, the field function differential equation solution is as follows:

$$\frac{\partial^2}{\partial y^2} \phi_1(\alpha, y) = \gamma_1^2 \phi_1(\alpha, y) = (\alpha^2 - k_{c_1}^2) \phi_1(\alpha, y) \quad (38)$$

$$(s^2 - \gamma_1^2) \phi_1(s) = 0. \quad (39)$$

Therefore,

$$\phi_1(\alpha, y) = R_1(\alpha) e^{+\gamma_1 y} + R_2(\alpha) e^{-\gamma_1 y}. \quad (40)$$

Clearly, for infinite positive values of  $y$ , the field should vanish, or,

$$\lim_{y \rightarrow \infty} \phi_1(\alpha, y) = 0 \quad (41)$$

then,

$$\phi_1(\alpha, y) = R_2(\alpha) e^{-\gamma_1 y} = A(\alpha) e^{-\gamma_1 y} \quad (42)$$

where



$$A(\alpha) = R_2(\alpha).$$

Similarly for Region 2, there are two solutions, corresponding to the real or imaginary character of  $\gamma_2$ . The solution for  $\gamma_2$  imaginary is

$$\Phi_2(\alpha, y) = B(\alpha) \text{Sin}^h \gamma_2 y + C(\alpha) \text{Cos}^h \gamma_2 y \quad (43)$$

where

$$\gamma_2 = j\gamma_2''$$

and for  $\gamma_2$  real it is

$$\Phi_2(\alpha, y) = B(\alpha) \text{Sinh} \gamma_2 y + C(\alpha) \text{Cosh} \gamma_2 y. \quad (44)$$

Therefore, the transforms of  $\phi_1^e$ ,  $\phi_1^h$ ,  $\phi_2^e$ ,  $\phi_2^h$  are given by

#### Region 1

$$\phi_1^e(\alpha, y) = A^e(\alpha) e^{-\gamma_1(y-D)} \quad (45)$$

$$\phi_1^h(\alpha, y) = A^h(\alpha) e^{-\gamma_1(y-D)} \quad (46)$$

#### Region 2

$$\phi_2^e(\alpha, y) = \begin{cases} B_H^e(\alpha) \text{Sinh} \gamma_2 y + C_H^e(\alpha) \text{Cosh} \gamma_2 y, & \gamma_2 \text{ Real} \end{cases} \quad (47)$$

$$\phi_2^e(\alpha, y) = \begin{cases} jB_T^e(\alpha) \text{Sin} \gamma_2'' y + C_T^e(\alpha) \text{Cos} \gamma_2'' y, & \gamma_2 \text{ Imaginary} \end{cases} \quad (48)$$

$$\phi_2^h(\alpha, y) = \begin{cases} B_H^h(\alpha) \text{Sinh} \gamma_2 y + C_H^h(\alpha) \text{Cosh} \gamma_2 y, & \gamma_2 \text{ Real} \end{cases} \quad (49)$$

$$\phi_2^h(\alpha, y) = \begin{cases} jB_T^h(\alpha) \text{Sin} \gamma_2'' y + C_{H_T}^h(\alpha) \text{Cos} \gamma_2'' y, & \gamma_2 \text{ Imaginary} \end{cases} \quad (50)$$



where superscript (e) denotes the electric field case and (h) magnetic field case and where

$$\gamma_2'' = -j\gamma_2.$$

Spectral Domain representation of the boundary condition expressions can be obtained by taking the Fourier transform of the equations (7) through (14) with the following results:

$$k_{c2}^2 \Phi_2^e(\alpha, 0) = 0 \quad (51) \checkmark$$

$$-j\alpha\gamma\Phi_2^e(\alpha, 0) - j\omega\mu_2 \frac{\partial \Phi_2^h}{\partial y}(\alpha, 0) = 0 \quad (52) \checkmark$$

$$k_{c1}^2 \Phi_1^e(\alpha, D) = k_{c2}^2 \Phi_2^e(\alpha, D) \quad (53) \checkmark$$

$$\begin{aligned} -j\alpha\gamma\Phi_1^e(\alpha, D) - j\omega\mu_1 \frac{\partial \Phi_1^h}{\partial y}(\alpha, D) &= -j\alpha\gamma\Phi_2^e(\alpha, D) \\ &- j\omega\mu_2 \frac{\partial \Phi_2^h}{\partial y}(\alpha, D) \end{aligned} \quad (54) \checkmark$$

$$k_{c1}^2 \Phi_1^e(\alpha, D) = E_z(\alpha) \quad (55) \checkmark$$

$$-j\alpha\gamma\Phi_1^e(\alpha, D) - j\omega\mu_1 \frac{\partial \Phi_1^h}{\partial y}(\alpha, D) = E_x(\alpha) \quad (56) \checkmark$$

$$k_{c1}^2 \Phi_1^h(\alpha, D) - k_{c2}^2 \Phi_2^h(\alpha, D) = J_x(\alpha) \quad (57) \checkmark$$

$$\begin{aligned} -j\alpha\gamma\Phi_1^h(\alpha, D) + j\omega\epsilon_1 \frac{\partial \Phi_1^e}{\partial y}(\alpha, D) + j\alpha\gamma\Phi_2^h(\alpha, D) \\ - j\omega\epsilon_2 \frac{\partial \Phi_2^e}{\partial y}(\alpha, D) = J_z(\alpha). \end{aligned} \quad (58) \checkmark$$

Where the derivative transform pair

$$F \left[ \frac{\partial \phi(x, y)}{\partial x} \right] = -j\alpha\phi(\alpha, y)$$

has been applied.



By substituting the Field expressions for real (hyperbolic case) and the imaginary (trigonometric case) values of  $\gamma_2$  into the transformed equations (51) through (58), the following equations are obtained.

Hyperbolic Case:

$$k_{c_2}^2 C_H^e(\alpha) = 0 \quad (59)$$

$$-j[\alpha \gamma C_H^e(\alpha) + \omega \mu_2 \gamma_2 B_H^h(\alpha)] = 0 \quad (60) \quad \checkmark$$

$$k_{c_1}^2 A^e(\alpha) = k_{c_2}^2 [C_H^e(\alpha) \cosh \gamma_2 D + B_H^e(\alpha) \sinh \gamma_2 D] \quad (61) \quad \checkmark$$

$$-j(\alpha \gamma A^e(\alpha) - \omega \mu_1 \gamma_1 A^h(\alpha)) = -j\alpha \gamma [B_H^e(\alpha) \sinh \gamma_2 D + C_H^e(\alpha) \cosh \gamma_2 D] - j\omega \mu_2 \gamma_2 [B_H^h(\alpha) \cosh \gamma_2 D + C_H^h(\alpha) \sinh \gamma_2 D] \quad (62)$$

$$k_{c_1}^2 A^e(\alpha) = E_z(\alpha) \quad (63) \quad \checkmark$$

$$-j(\alpha \gamma A^e(\alpha) - \omega \mu_1 \gamma_1 A^h(\alpha)) = E_x(\alpha) \quad (64) \quad \checkmark$$

$$k_{c_1}^2 A^h(\alpha) - k_{c_2}^2 [B_H^h(\alpha) \sinh \gamma_2 D + C_H^h(\alpha) \cosh \gamma_2 D] = J_x(\alpha) \quad (65) \quad \checkmark$$

$$-j(\alpha \gamma A^h(\alpha) + \omega \epsilon_1 \gamma_1 A^e(\alpha)) + j\alpha \gamma [B_H^h(\alpha) \sinh \gamma_2 D + C_H^h(\alpha) \cosh \gamma_2 D] - j\omega \epsilon_2 \gamma_2 [B_H^e(\alpha) \cosh \gamma_2 D + C_H^e(\alpha) \sinh \gamma_2 D] = J_z(\alpha). \quad (66) \quad \checkmark$$

Similarly for imaginary values of  $\gamma_2$  by making the substitutions

$$\begin{aligned} \gamma_2 &= j\gamma_2'' \\ \sinh \gamma_2 D &= j \sin \gamma_2'' D \end{aligned} \quad (67)$$



$$\text{Cosh}\gamma_2 D = \text{Cos}\gamma_2'' D$$

in equations (59) through (66) a corresponding set for the trigonometric case can be obtained.

Finally constants for electric and magnetic field expressions are determined as follows:

$$A^e(\alpha) = \frac{1}{k_{c1}^2} E_z(\alpha) \quad (68)$$

$$A^h(\alpha) = \frac{1}{j\omega\mu_1\gamma_1} E_x(\alpha) + \frac{\alpha\gamma}{\omega\mu_1\gamma_1 k_{c1}^2} E_z(\alpha) \quad (69)$$

$$C_{T,H}^e(\alpha) = 0 \quad (70)$$

$$B_{T,H}^h(\alpha) = 0 \quad (71)$$

$$B_T^e(\alpha) = \frac{1}{jk_{c2}^2 \text{Sin}\gamma_2'' D} E_z(\alpha) \quad (72)$$

$$B_H^e(\alpha) = \frac{1}{k_{c2}^2 \text{Sinh}\gamma_2 D} E_z(\alpha) \quad (73)$$

$$C_H^h(\alpha) = - \frac{1}{\omega\mu_2\gamma_2 \text{Sinh}\gamma_2 D} \left[ \frac{\alpha\gamma}{k_{c2}^2} E_z(\alpha) - jE_x(\alpha) \right] \quad (74)$$

$$C_T^h(\alpha) = \frac{1}{\omega\mu_2\gamma_2 \text{Sin}\gamma_2'' D} \left[ \frac{\alpha\gamma}{k_{c2}^2} E_z(\alpha) - jE_x(\alpha) \right] \quad (75)$$

The subscript T denotes the trigonometric case and H denotes the Hyperbolic case. Substituting equations (70) through (75) into equation (65) and (66) one can obtain two sets of equations of the form



$$F_1(\alpha, \beta) E_x(\alpha) + F_2(\alpha, \beta) E_z(\alpha) = J_x(\alpha) \quad (76)$$

$$F_3(\alpha, \beta) E_x(\alpha) + F_4(\alpha, \beta) E_z(\alpha) = J_x(\alpha) \quad (77)$$

where

$$\gamma = j\beta.$$

For the hyperbolic case the  $F_i$  have the form

$$F_{1H}(\alpha, \beta) = -j \left[ \frac{k_{c1}^2}{\omega\mu_1\gamma_1} + \frac{k_{c2}^2}{\omega\mu_2\gamma_2} \text{Cotanh}\gamma_2 D \right] \quad (78)$$

$$F_{2H}(\alpha, \beta) = j \left[ \frac{\alpha\beta}{\omega\mu_1\gamma_1} + \frac{\alpha\beta}{\omega\mu_2\gamma_2} \text{Cotanh}\gamma_2 D \right] \quad (79)$$

$$F_{3H}(\alpha, \beta) = -F_{2H}(\alpha, \beta) \quad (80)$$

$$F_{4H}(\alpha, \beta) = -j \left[ -\frac{(\alpha\beta)^2}{\omega\mu_1\gamma_1 k_{c1}^2} + \frac{\omega\epsilon_1\gamma_1}{k_{c1}^2} - \frac{\text{Cotanh}\gamma_2 D}{k_{c2}^2} \right. \\ \left. \left( \frac{(\alpha\beta)^2}{\omega\mu_2\gamma_2} - \omega\epsilon_2\gamma_2 \right) \right] \quad (81)$$

A similar set of equations for the trigonometric case can be obtained by use of equation (67). Solutions of equations (76) and (77) for  $E_z(\alpha)$  and  $E_x(\alpha)$  in terms of  $J_z(\alpha)$  and  $J_x(\alpha)$  may be obtained as follows:

$$E_x(\alpha) = \frac{F_4(\alpha, \beta) J_x(\alpha) - F_2(\alpha, \beta) J_z(\alpha)}{D_N} \quad (82)$$



$$E_z(\alpha) = \frac{-F_3(\alpha, \beta)J_x(\alpha) + F_1(\alpha, \beta)J_z(\alpha)}{D_N} \quad (83)$$

where

$$D_N = F_1(\alpha, \beta)F_4(\alpha, \beta) - F_2(\alpha, \beta)F_3(\alpha, \beta). \quad (84)$$

Define the following constants,

$$G_1(\alpha, \beta) = \frac{F_4(\alpha, \beta)}{D_N} \quad (85)$$

$$G_2(\alpha, \beta) = \frac{-F_2(\alpha, \beta)}{D_N} \quad (86)$$

$$G_3(\alpha, \beta) = \frac{-F_3(\alpha, \beta)}{D_N} \quad (87)$$

$$G_4(\alpha, \beta) = \frac{F_1(\alpha, \beta)}{D_N} \quad (88)$$

finally obtaining

$$E_x(\alpha) = G_1(\alpha, \beta)J_x(\alpha) + G_2(\alpha, \beta)J_z(\alpha) \quad (89)$$

$$E_z(\alpha) = G_3(\alpha, \beta)J_x(\alpha) + G_4(\alpha, \beta)J_z(\alpha). \quad (90)$$

### C. PHYSICAL PARAMETERS

General equations (89) and (90) show no dependence on the actual physical configuration other than the boundary conditions.

In general, the transformed surface current densities  $J_x(\alpha)$  and  $J_z(\alpha)$  can be expanded in a set of known basis functions as,



$$J_x(\alpha) = \sum_{i=1}^{\infty} a_i j_{xi}(\alpha) \quad (91)$$

$$J_z(\alpha) = \sum_{i=1}^{\infty} b_i j_{zi}(\alpha). \quad (92)$$

Since the two conductors are sufficiently narrow, one may assume that the surface current density in the x-direction is zero. By modifying equations (89) and (90), also substituting (91) and (92) one obtains

$$G_2(\alpha, \beta) \sum_{i=1}^{\infty} b_i j_{zi}(\alpha) = E_x(\alpha) \quad (93)$$

$$G_4(\alpha, \beta) \sum_{i=1}^{\infty} b_i j_{zi}(\alpha) = E_z(\alpha). \quad (94)$$

Furthermore, although one could perform a multiterm approximation to the surface current density in the z-direction, in this case a one term approximation was found sufficient after different distributions were tried. For the numerical results of this present work uniform surface current density is used, thus

$$G_2(\alpha, \beta) b_1 j_{zi}(\alpha) = E_x(\alpha) \quad (95)$$

$$G_4(\alpha, \beta) b_1 j_{zi}(\alpha) = E_x(\alpha), \quad (96)$$

or substituting equation (92) for the case where  $i = 1$ ,

$$G_2(\alpha, \beta) J_z(\alpha) = E_x(\alpha) \quad (97)$$

$$G_4(\alpha, \beta) J_z(\alpha) = E_z(\alpha). \quad (98)$$



It is clear that one only needs to work with either one of the above equations. Assuming a uniform current distribution

$$J_z(x) = \begin{cases} I_0/W & \text{on the strip} \\ 0 & \text{elsewhere} \end{cases}$$

where  $W$  is the width of the strip and  $I_0$  is the magnitude of the current, one can obtain the  $\alpha$ -Domain Fourier transform,  $J_z(\alpha)$ . This was done for three different configurations, namely single strip, coupled strips even mode and coupled strips odd mode.

For the geometry in Figure 3 one obtains the following:

Single Strip:

$$J_z(\alpha) = \frac{I_0}{W} \int_{-W/2}^{W/2} e^{j\alpha x} dx, \quad J_z(\alpha) = I_0 \frac{\sin(\frac{\alpha W}{2})}{\frac{\alpha W}{2}} \quad (99)$$

Coupled Strips, Even Mode:

$$J_z(\alpha) = \frac{I_0}{W} \int_{-(S/2+W)}^{-S/2} e^{j\alpha x} dx + \int_{S/2}^{(S/2+W)} e^{j\alpha x} dx \quad (100)$$

$$J_z(\alpha) = 2I_0 \cos \frac{\alpha(S+W)}{2} \frac{\sin(\frac{\alpha W}{2})}{(\frac{\alpha W}{2})}$$

Odd Mode:

$$J_z(\alpha) = \frac{I_0}{W} \int_{-(S/2+W)}^{-S/2} e^{j\alpha x} dx - \int_{S/2}^{-(S/2+W)} e^{j\alpha x} dx \quad (101)$$



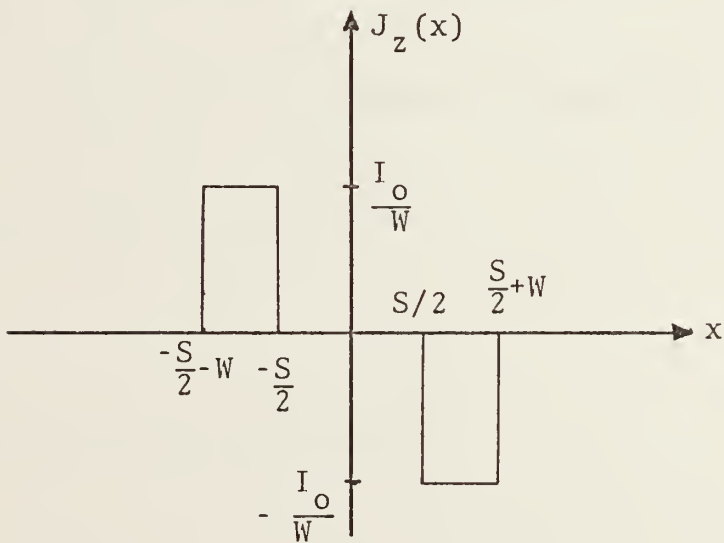
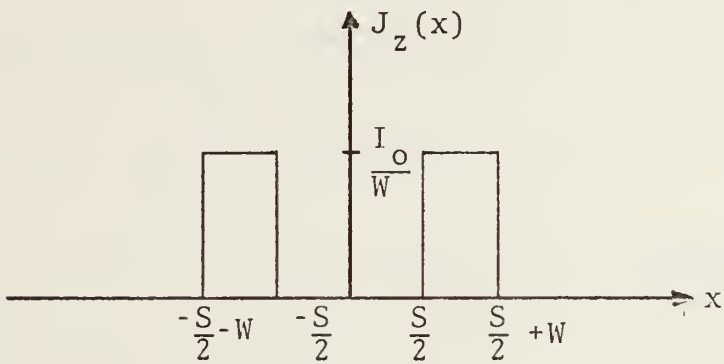
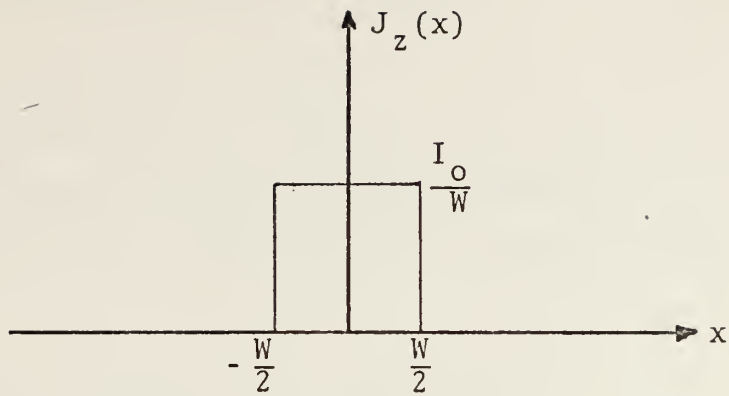


Figure 3. Single and Coupled Strips Even and Odd Longitudinal Current Distributions.



$$J_z(\alpha) = j2I_0 \sin \frac{\alpha(S+W)}{2} \frac{\sin(\frac{\alpha W}{2})}{(\frac{\alpha W}{2})}. \quad (101)$$

It is convenient to make equation (98) independent of  $E_z(\alpha)$ . This can be done by taking an inner product with a function orthogonal to  $E_z(\alpha)$ . Applying this concept, equation (98) becomes

$$\langle G_4(\alpha, \beta) J_z(\alpha), W(\alpha) \rangle = \langle E_z(\alpha), W(\alpha) \rangle \quad (102)$$

where the inner product is defined by

$$\langle Y(\alpha), W(\alpha) \rangle = \int_{-\infty}^{+\infty} Y(\alpha) W(\alpha) d\alpha. \quad (103)$$

A suitable weighting function  $W(\alpha)$  is the complex conjugate of  $J_z(\alpha)$ ; i.e.  $J_z^*(\alpha)$  or  $J_z(-\alpha)$ .

Then by Parseval's theorem, the right-hand-side of equation (98) becomes zero because of the orthogonality of the  $E_z(\alpha)$  and  $J_z(\alpha)$ .

Therefore equation (98) takes the final form

$$\int_{-\infty}^{+\infty} G_4(\alpha, \beta) J_z^*(\alpha) J_z(\alpha) d\alpha = 0 \quad (104)$$

or

$$\int_{-\infty}^{+\infty} G_4(\alpha, \lambda/\lambda') |J_z(\alpha)|^2 d\alpha = 0 \quad (105)$$

where

$$\beta = 2\pi/\lambda'.$$



So, one can see the integrand is dependent on the ratio of  $\lambda/\lambda'$  or effective dielectric constant,  $\epsilon_{r_{eff}}$ , defined by

$$\epsilon_{r_{eff}} = \left( \frac{\lambda}{\lambda'} \right)^2.$$

Using the preceding current distributions for single and coupled lines, the phase velocity characteristics can be determined by finding those values of the propagation constant that give a zero value for the integral at a particular frequency.

It is clear that once equation (105) is integrated the dependence on the variable  $\alpha$  disappears, so one can state that the whole process is really a function of frequency and the structure's physical characteristics, namely, width of the strips  $W$ , and in the case of coupled strips, separation  $S$  between the strips, thickness of substrate  $D$ , and the dielectric's relative permittivity  $\epsilon_r$ .

By observing the behavior of each term equation (105) one can conclude integration of the  $|J_z(\alpha)|^2$  term always gives a positive area. In the three configuration of microstrip which were studied, the Fourier transforms of the current distributions always provided a multiplicative factor of  $(\text{Sin}\alpha/\alpha)^2$  which gives rise to a curve having a large main lobe and low side lobes in the  $\alpha$ -domain.

Several plots of  $G_4(\alpha, \lambda/\lambda')$  and the  $G_4(\alpha, \lambda/\lambda') \cdot |J_z(\alpha)|^2$  term between limits of  $\alpha$ ;  $-3000 < \alpha < 3000$  for different  $\lambda/\lambda'$  values is shown in Figure 4 and Figure 5 respectively.



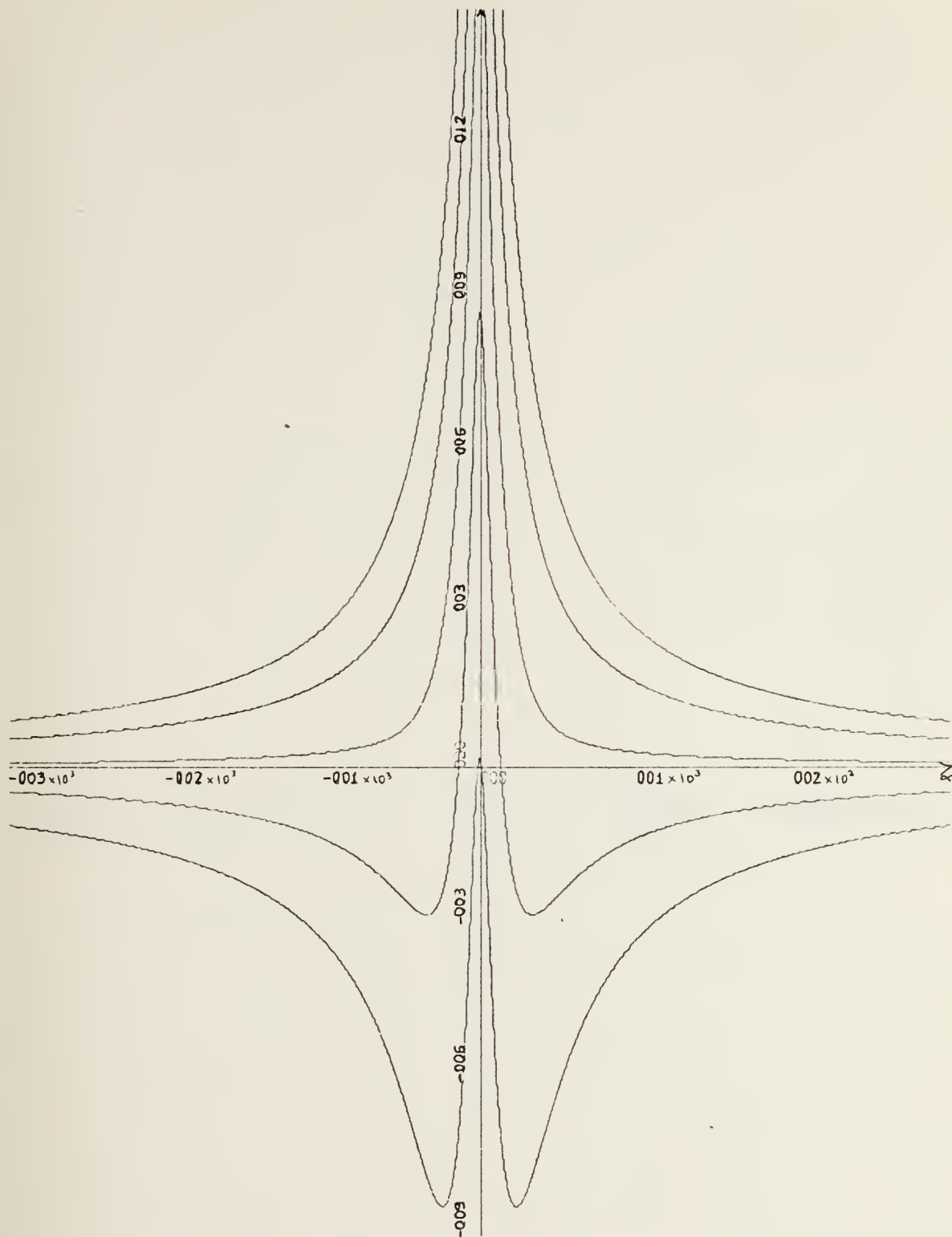


Figure 4.  $G_4(\alpha, \lambda/\lambda')$  versus  $\alpha$ .



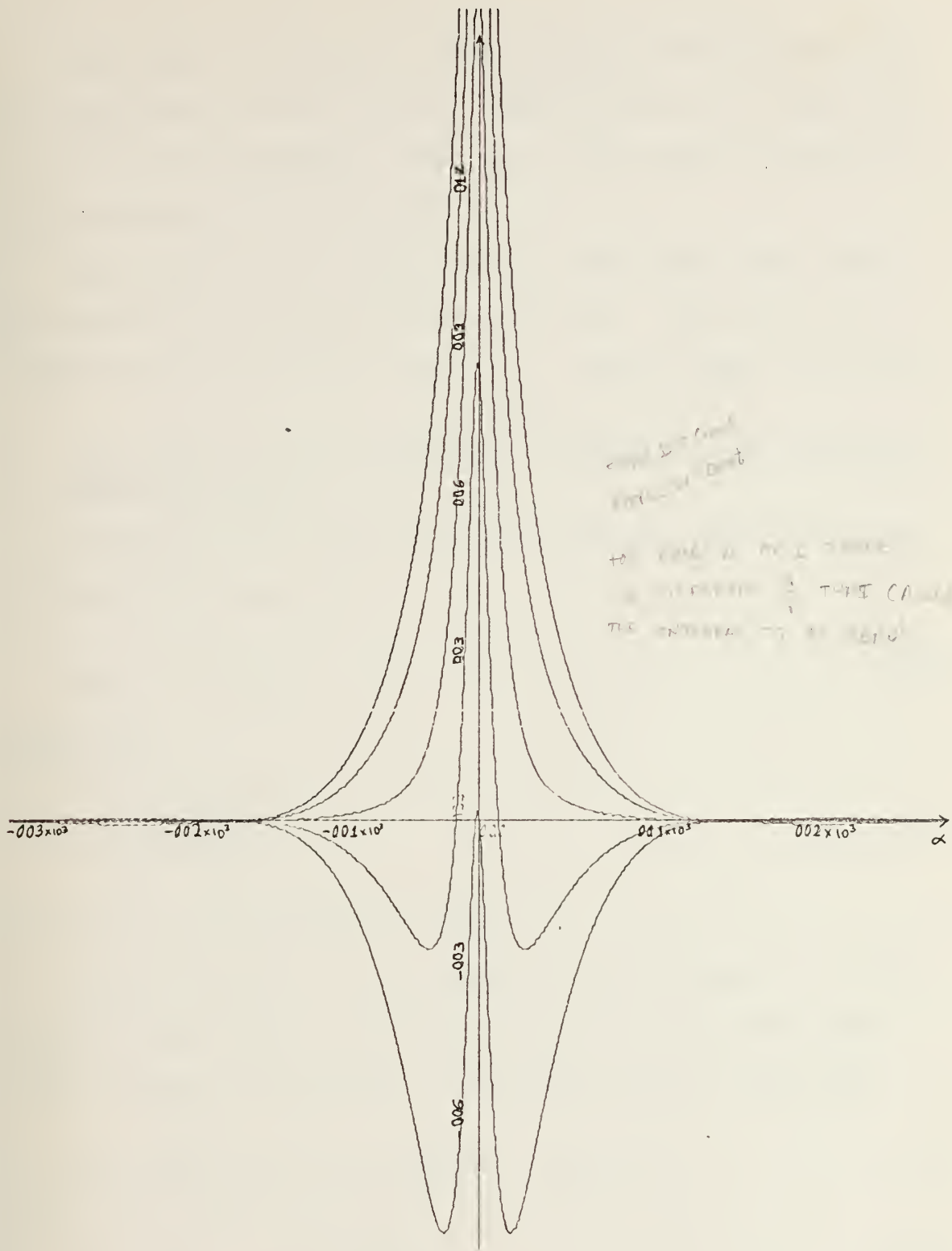


Figure 5.  $G_4(\alpha, \lambda/\lambda') |J_z(\alpha)|^2$  versus  $\alpha$ .



The family of curves is symmetrical to the y-axis and for decreasing values of  $\lambda/\lambda'$ , moves along the y-axis without losing the symmetry. The total area under it monotonically decreases.

The value of  $\lambda/\lambda'$  which gives a zero total area under the curve is the root of equation (105). This gives the effective dielectric constant  $\epsilon_{r_{\text{eff}}}$  and the phase velocity.

#### D. THE CHARACTERISTIC IMPEDANCE IN TERMS OF THE DISPERSION CHARACTERISTICS

In general, one can express the average power in terms of the strip current as,

$$P_{\text{AVE}} = \frac{1}{2} I_{0_z}^2 Z_0 \quad (106)$$

from which

$$Z_0 = \frac{2P_{\text{AVE}}}{I_{0_z}^2}, \quad (107)$$

provided  $I_0$  is a unique current.

In this study, it was assumed in the dispersion characteristic part that the surface current across the strips were uniform; furthermore, the current can be expressed, as

$$I_{0_z} = \oint \vec{H} \cdot d\vec{\ell} = \int_{\text{STRIP}} (H_{x_1} - H_{x_2}) dx \quad (108)$$

$$I_{0_z} = \int_{\text{STRIP}} |J_z(x)| dx. \quad (109)$$

In the case of a single strip,



$$J_z(x) = \begin{cases} \frac{I_0}{W} & \text{on the strips} \\ 0 & \text{elsewhere.} \end{cases}$$

For simplicity  $I_0$  is arbitrarily set equal to unity. Thus,

$$I_{0z}^2 = 1. \quad (110)$$

As stated by Collin [Ref. 1], a general expression for the time average power flow in terms of the electric and magnetic fields is

$$P_{AVE}(x,y) = \text{Re} \left[ \frac{1}{2} \iint_S \bar{E} \times \bar{H}^* \cdot \bar{a}_z da \right] \quad (111)$$

where

$$\bar{E} \times \bar{H}^* \cdot \bar{a}_z = E_x H_y^* - E_y H_x^*.$$

This equation becomes

$$P_{AVE}(x,y) = \text{Re} \left[ \frac{1}{2} \iint_S (E_x H_y^* - E_y H_x^*) dx dy \right] \quad (112)$$

Referring back to equations (1) through (6), it is found that the equations for the electric and magnetic fields can be in terms of Hertzian potential functions.

So substituting equations (3) through (6) in the expression for average power,

$$P_{AVE}(x,y) = \frac{1}{2} \text{Re} \left\{ \iint_S \left( j\beta \frac{\partial \phi^e}{\partial x} - j\omega\mu \frac{\partial \phi^h}{\partial y} \right) e^{j\beta z} \left( -j\beta \frac{\partial \phi^h}{\partial y} + j\omega\epsilon \frac{\partial \phi^e}{\partial x} \right) e^{-j\beta z} dx dy + \iint_S \left( j\beta \frac{\partial \phi^e}{\partial y} + j\omega\mu \frac{\partial \phi^h}{\partial x} \right) e^{j\beta z} \right. \quad (113)$$



$$\left. (-j\beta \frac{\partial \phi}{\partial x} - j\omega\epsilon \frac{\partial \phi^e}{\partial y}) e^{-j\beta z} dx dy \right\}. \quad (113)$$

Applying Parseval's theorem

$$P_{AVE} = \frac{1}{4\pi} \operatorname{Re} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ \left( \alpha\beta\phi^e(\alpha, y) - j\omega\mu \frac{\partial \phi^h(\alpha, y)}{\partial y} \right) \right. \right. \\ \left. \left( -j\beta \frac{\partial \phi^h(\alpha, y)}{\partial y} - \alpha\omega\epsilon \phi^{e*}(\alpha, y) \right) - \left( j\beta \frac{\partial \phi^e(\alpha, y)}{\partial y} + \alpha\omega\mu \phi^h(\alpha, y) \right) \right. \\ \left. \left. \left( \beta\alpha\phi^{h*}(\alpha, y) - j\omega\epsilon \frac{\partial \phi^{e*}(\alpha, y)}{\partial y} \right) \right] d\alpha dy \right\}. \quad (114)$$

where the Fourier transform of  $\partial\phi/\partial x$  has been applied from equation (114). Performing the operations indicated,

$$P_{AVE} = \frac{1}{4\pi} \operatorname{Re} \left\{ \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ -\alpha^2\beta\omega\epsilon |\phi^e(\alpha, y)|^2 - \omega\beta\mu \left| \frac{\partial \phi^h(\alpha, y)}{\partial y} \right|^2 \right. \right. \\ - \omega\beta\epsilon \left| \frac{\partial \phi^e(\alpha, y)}{\partial y} \right|^2 - \alpha^2\omega\mu\beta |\phi^h(\alpha, y)|^2 \\ + j\alpha k^2 \left( \phi^h(\alpha, y) \frac{\partial \phi^{e*}(\alpha, y)}{\partial y} + \phi^{e*}(\alpha, y) \frac{\partial \phi^h(\alpha, y)}{\partial y} \right) \\ \left. \left. - j\alpha\beta^2 \left( \phi^e(\alpha, y) \frac{\partial \phi^{h*}(\alpha, y)}{\partial y} + \phi^{h*}(\alpha, y) \frac{\partial \phi^e(\alpha, y)}{\partial y} \right) \right] d\alpha dy \right\} \quad (115)$$

where

$$k^2 = \omega^2\mu\epsilon.$$

After obtaining the general expression for average power, one can apply it to both regions for further specific equations.



In Region 1,

$$\Phi_1^e(\alpha, y) = A^e(\alpha) e^{-\gamma_1(y-D)} \quad (116)$$

$$\Phi_1^h(\alpha, y) = A^h(\alpha) e^{-\gamma_1(y-D)} \quad (117)$$

$$\frac{\partial \Phi_1^e(\alpha, y)}{\partial y} = -\gamma_1 A^e(\alpha) e^{-\gamma_1(y-D)} \quad (118)$$

$$\frac{\partial \Phi_1^h(\alpha, y)}{\partial y} = -\gamma_1 A^h(\alpha) e^{-\gamma_1(y-D)} \quad (119)$$

Substituting the preceding equations in (115) one can obtain

$$P_{1AVE} = \frac{1}{4\pi} \operatorname{Re} \left\{ \int_{-\infty}^{+\infty} \int_D^{+\infty} e^{-2\gamma_1(y-D)} \left[ -\alpha^2 \mu \omega \epsilon_1 |A^e(\alpha)|^2 \right. \right. \\ \left. \left. - \omega \beta \mu_1 \gamma_1^2 |A^h(\alpha)|^2 - \omega \beta \epsilon_1 \gamma_1^2 |A^e(\alpha)|^2 - \alpha^2 \omega \mu_1 \beta |A^h(\alpha)|^2 \right. \right. \\ \left. \left. - j\alpha k_1^2 (2\gamma_1 A^h(\alpha) A^{e*}(\alpha)) + j\alpha \beta^2 (2\gamma_1 A^e(\alpha) A^{h*}(\alpha)) \right] d\alpha dy \right\}. \quad (120)$$

It is clear from the above expression that one can integrate with respect to  $y$  to obtain

$$P_{1AVE} = -\frac{1}{8\pi} \int_{-\infty}^{+\infty} \left[ \left( \frac{\alpha^2 + \gamma_1^2}{\gamma_1^2} \right) \left( \beta \omega \epsilon_1 |A^e(\alpha)|^2 \right. \right. \\ \left. \left. + \beta \omega \mu_1 |A^h(\alpha)|^2 + 2\alpha(\beta^2 + k_1^2) \operatorname{Re} \left\{ -jA^e(\alpha) A^{h*}(\alpha) \right\} \right) \right] d\alpha. \quad (121)$$

In Region 2, clearly there are two expressions for average power. When  $\gamma_2$  is real, equations (47) and (49) apply as stated earlier.



$C_{T,H}(\alpha)$  and  $B_{T,H}^h(\alpha)$  are zero so,

$$\Phi_2^e(\alpha, y) = B_H^e(\alpha) \sinh \gamma_2 y \quad (122)$$

$$\Phi_2^h(\alpha, y) = C_H^h(\alpha) \cosh \gamma_2 y \quad (123)$$

$$\frac{\partial \Phi_2^h(\alpha, y)}{\partial y} = \gamma_2 C_H^h(\alpha) \sinh \gamma_2 y \quad (124)$$

$$\frac{\partial \Phi_2^e(\alpha, y)}{\partial y} = \gamma_2 B_H^e(\alpha) \cosh \gamma_2 y \quad (125)$$

and

$$\begin{aligned} P_{2AVE_H} = & - \frac{1}{4\pi} \operatorname{Re} \left\{ \int\limits_{\substack{\text{HYP} \\ \text{REGION}}}^D \int_0^D \sinh^2 \gamma_2 y [\alpha^2 \beta \omega \epsilon_2 |B_H^e(\alpha)|^2 \right. \\ & + \beta \omega \mu_2 \gamma_2^2 |C_H^h(\alpha)|^2 + j\alpha \beta^2 \gamma_2 B_H^e(\alpha) C_H^{h*}(\alpha) \\ & - j\alpha \gamma_2 k_2^2 C_H^h(\alpha) B_H^{e*}(\alpha)] d\alpha dy + \cosh^2 \gamma_2 y \\ & [\alpha^2 \beta \omega \mu_2 |C_H^h(\alpha)|^2 + \beta \omega \epsilon_2 \gamma_2^2 |B_H^e(\alpha)|^2 \\ & \left. + j\alpha \beta^2 \gamma_2 B_H^e(\alpha) C_H^{h*}(\alpha) - j\alpha k_2^2 \gamma_2 C_H^h(\alpha) B_H^{e*}(\alpha)] d\alpha dy \right\}. \end{aligned} \quad (126)$$

The  $y$ -dependence of the average power in Region 2 disappears through corresponding integrations as indicated below:

$$\int_0^D \sinh^2 \gamma_2 y dy = \frac{\sinh 2\gamma_2 D - 2\gamma_2 D}{2\gamma_2} \quad (127)$$



$$\int_0^D \cosh^2 \gamma_2 y dy = \frac{\sinh 2\gamma_2 D + 2\gamma_2 D}{2\gamma_2} . \quad (128)$$

Thus ,

$$P_{2\text{AVE}_H} = - \frac{1}{16\pi} \operatorname{Re} \left\{ \int_{\text{HYP REGION}} (\sinh 2\gamma_2 D - 2\gamma_2 D) \right. \\ \left[ \frac{\alpha^2 \beta \omega \epsilon_2}{\gamma_2} |B_H^e(\alpha)|^2 + \beta \omega \mu_2 \gamma_2 |C_H^h(\alpha)|^2 \right. \\ \left. + j\alpha \beta^2 B_H^e(\alpha) C_H^{*h}(\alpha) - j\alpha k_2^2 C_H^h(\alpha) B_H^{*e}(\alpha) \right] d\alpha \\ + \int_{\text{HYP REGION}} (\sinh 2\gamma_2 D + 2\gamma_2 D) \left[ \frac{\alpha^2 \beta \omega \mu_2}{\gamma_2} |C_H^h(\alpha)|^2 \right. \\ \left. + \beta \omega \epsilon_2 \gamma_2 |B_H^e(\alpha)|^2 + j\alpha \beta^2 B_H^e(\alpha) C_H^{*h}(\alpha) \right. \\ \left. - j\alpha k_2^2 C_H^h(\alpha) B_H^{*e}(\alpha) \right] d\alpha \left. \right\} . \quad (129)$$

It is also clear that for imaginary  $\gamma_2$  the average power expression can easily be obtained with substitution of (48) and (50) and the corresponding trigonometric case electric and magnetic field constants i.e. equation (72) and (74). A detailed exposure to such analysis is given in Appendix C.

Then for imaginary  $\gamma_2$  Region 2:



$$\begin{aligned}
P_{2\text{AVE}_T} = & - \frac{1}{16\pi} \operatorname{Re} \left\{ \int_{\text{TRIG REGION}} (2\gamma_2''D - \sin 2\gamma_2''D) \left[ \frac{\alpha^2 \beta \omega \epsilon_2}{\gamma_2''} \right. \right. \\
& |B_T^e(\alpha)|^2 + \beta \omega \mu_2 \gamma_2'' |C_T^h(\alpha)|^2 + \alpha \beta^2 B_T^e(\alpha) C_T^{*h}(\alpha) \\
& + \alpha k_2^2 C_T^h(\alpha) B_T^{*e}(\alpha) \left. \right] d\alpha + \int_{\text{TRIG REGION}} (2\gamma_2''D + \sin 2\gamma_2''D) \\
& \left. \left[ \frac{\alpha^2 \beta \omega \mu_2}{\gamma_2''} |C_T^h(\alpha)|^2 + \beta \omega \epsilon_2 \gamma_2'' |B_T^e(\alpha)|^2 - \alpha \beta^2 B_T^e(\alpha) C_T^{*h}(\alpha) \right. \right. \\
& \left. \left. - \alpha k_2^2 C_T^h(\alpha) B_T^{*e}(\alpha) \right] d\alpha \right\}. \tag{130}
\end{aligned}$$

Total power in Region 2 becomes:

$$P_{2\text{AVE}} = P_{2\text{AVE}_H} + P_{2\text{AVE}_T}. \tag{131}$$

As strip width decreases one observes a widening  $\alpha$ -domain power distribution in Region 1. Also for higher values of dielectric constant where the energy is more concentrated into the substrate, a wider  $\alpha$ -domain power distribution is obtained in Region 2.

Finally, characteristic impedance can be found as

$$Z_0 = \frac{2(P_{1\text{AVE}} + P_{2\text{AVE}})}{I_{0z}^2} \tag{132}$$

In all cases, geometry is taken into account in the current distribution expressions.

Plots of  $\alpha$ -domain power distribution for Regions 1 and 2 are shown in Figure 6 and Figure 7 respectively. By



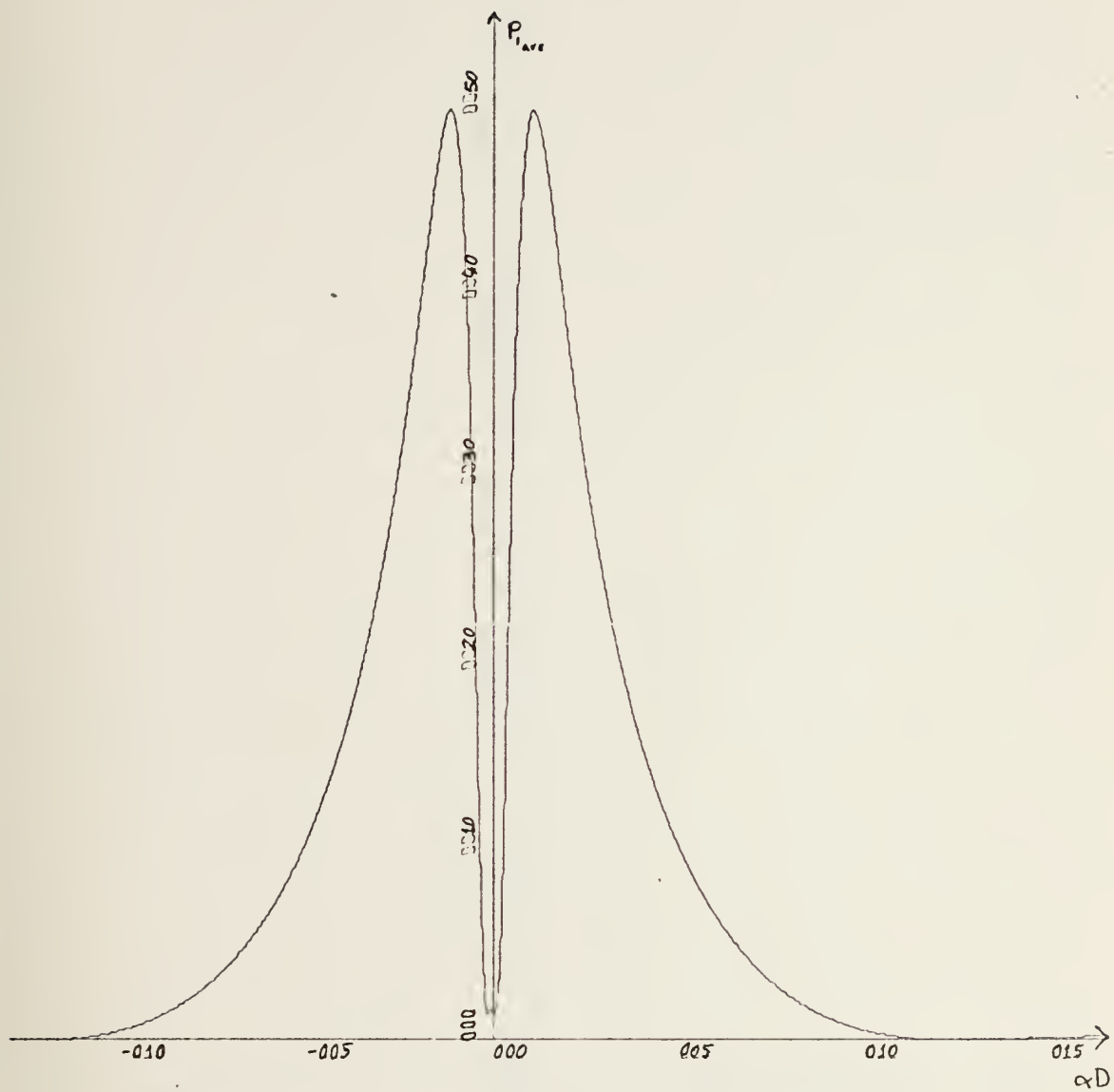


Figure 6. Region 1 Average Power Distribution in  $\alpha$ -Domain.



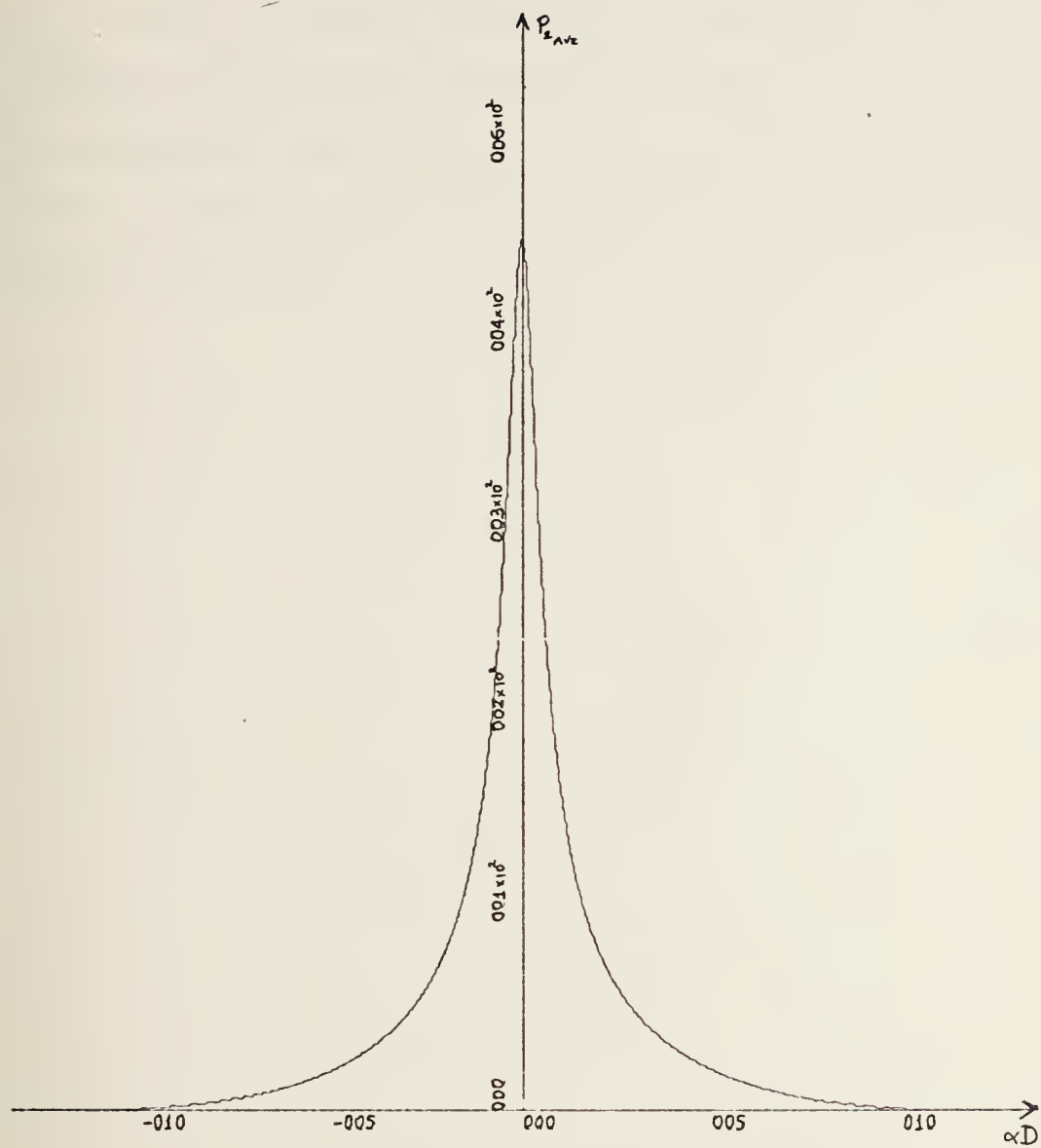


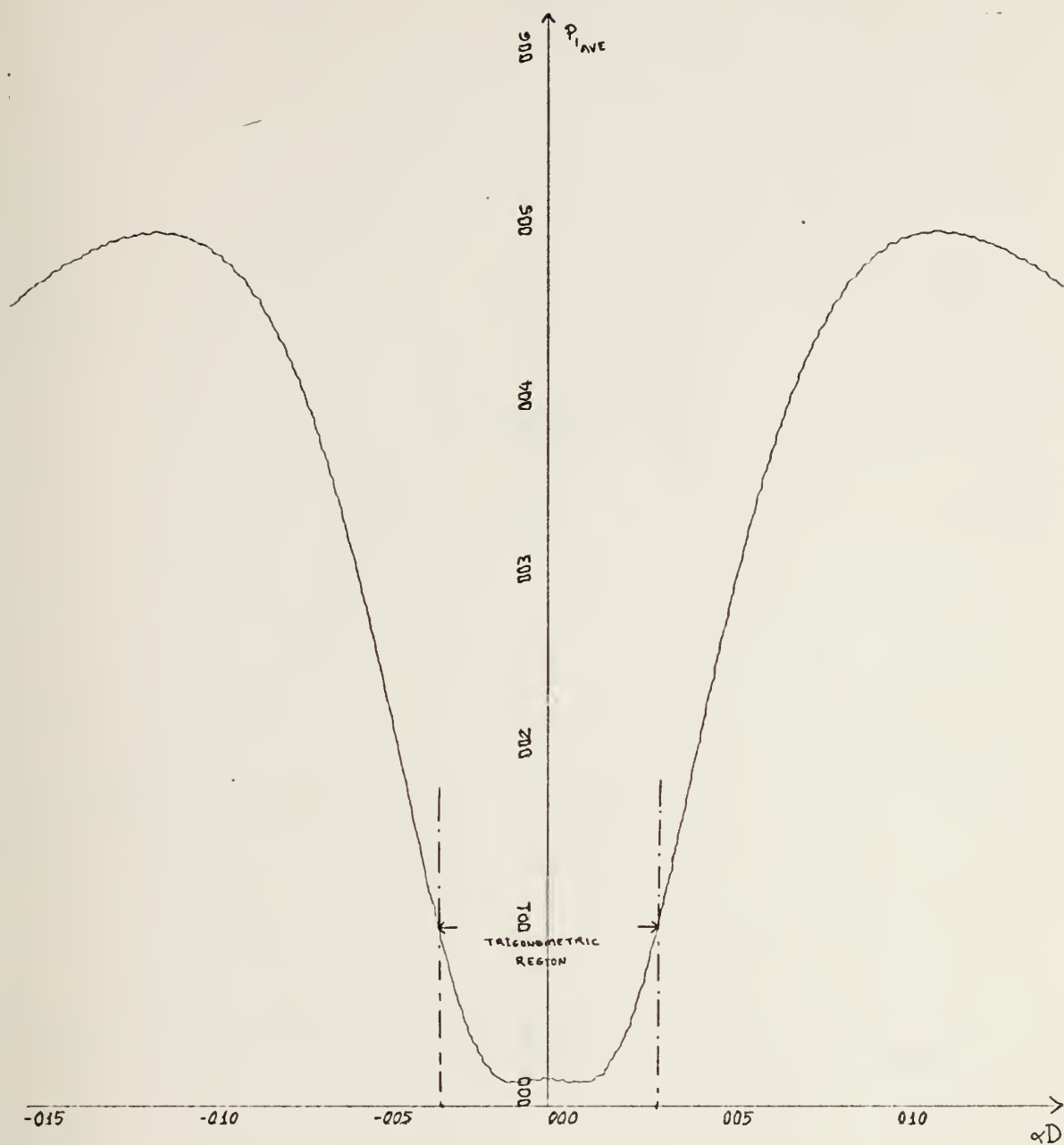
Figure 7. - Region 2 Average Power Distribution in  $\alpha$ -Domain.



taking the inverse Fourier transforms of  $P_{1_{AVE}}$  and  $P_{2_{AVE}}$  one could obtain the x-domain power distribution.

It is important to observe and prove the continuation of  $P_{1_{AVE}}$  and  $P_{2_{AVE}}$  curves for the critical points where  $\gamma_2$  switches from real to imaginary character or vice versa. The enlarged views of  $P_{1_{AVE}}$  and  $P_{2_{AVE}}$  given in Figures 8 and 9 clearly show the smoothness of transition from one region to another.

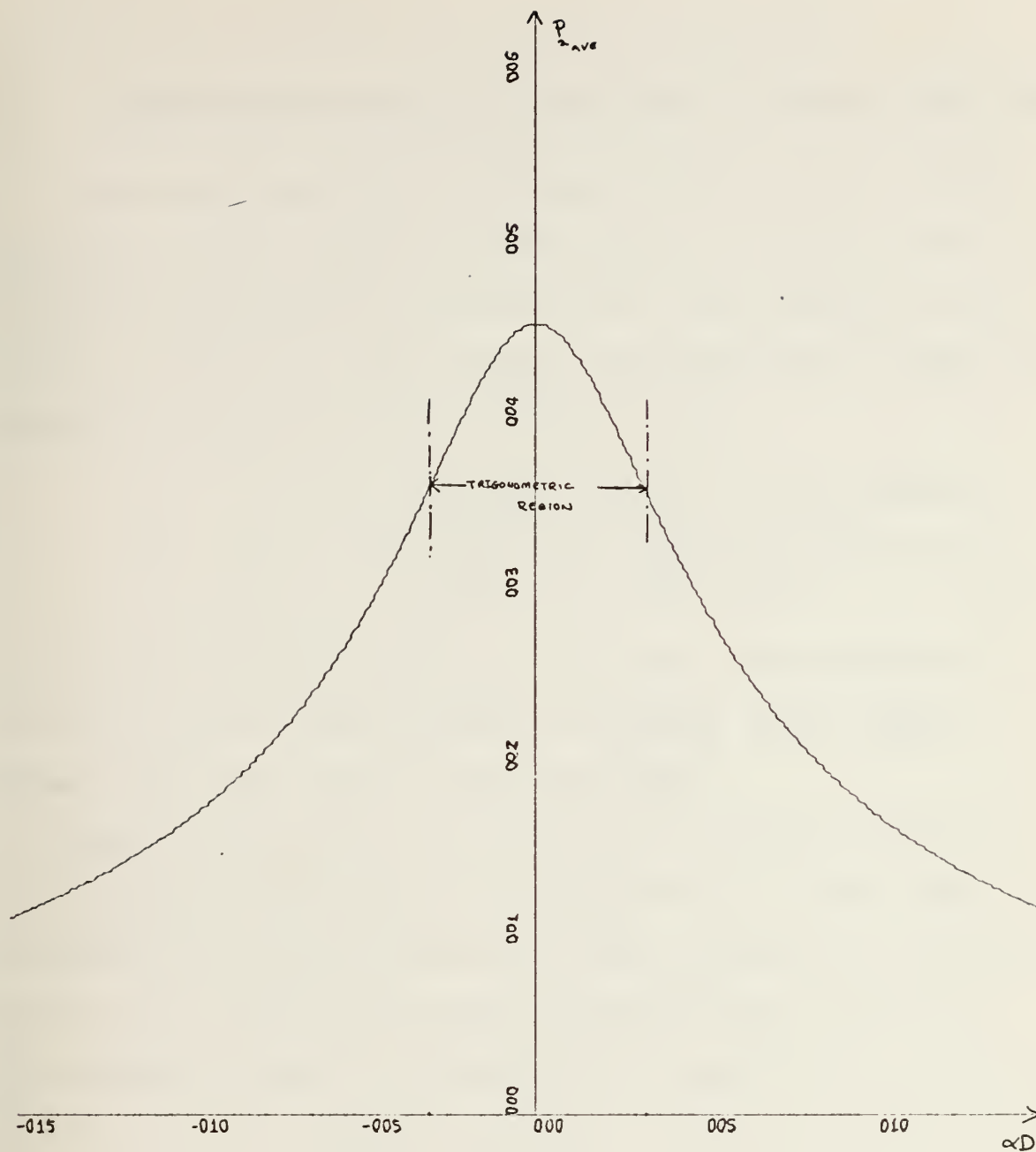




X-SCALE =  $5.00E-01$  UNITS INCH.  
 Y-SCALE =  $1.00E+01$  UNITS INCH.

Figure 8.  $P_{1\_AVE}$  versus  $\alpha D$  at 1 GHz.





X-SCALE =  $5.00E-01$  UNITS INCH

Y-SCALE =  $1.00E+02$  UNITS INCH

Figure 9.  $P_{2_{AVE}}$  versus  $\alpha D$  at 1 GHz.



### III. PERTURBATION ANALYSIS OF MICROSTRIP ON FERRITE SUBSTRATE

Considered here is an extension of the spectral domain technique to the analysis of microstrip on a ferrite substrate. A familiar perturbation expression for the propagation constant of a hollow, closed boundary waveguide with a change in material is given by Helszajn [Ref. 5] as:

$$(\Gamma' + \Gamma^*) = \frac{j\omega \iint_S (\epsilon_o [\Delta\chi_e] \cdot \bar{E}' \cdot \bar{E}^* + \mu_o [\Delta\chi_m] \cdot \bar{H}' \cdot \bar{H}^*) da}{\iint_S (\bar{E}^* \times \bar{H}' + \bar{E}' \times \bar{H}^*) \cdot \bar{a}_z da} \quad (133)$$

which primes denote perturbed quantities. This expression also is valid for open boundary structures which support bound waves [Ref. 7] as is the case here.

Consider the case of a microstrip on ferrite with magnetization perpendicular to the substrate. In this case, assuming the lossless dielectric substrate problem is solved, there is no change in electric susceptibility so  $[\Delta\chi_e] = 0$ . The change in magnetic susceptibility is given by

$$[\Delta\chi_m] = \begin{bmatrix} \chi_{xx} & 0 & \chi_{xz} \\ 0 & 0 & 0 \\ \chi_{zx} & 0 & \chi_{zz} \end{bmatrix} \quad (134)$$

where magnetic loss is permitted. The denominator of (133) is just  $4P_{AVG}$  so one obtains

$$\Gamma' + \Gamma^* = j\omega\mu_o \frac{\iint_S [\Delta\chi_m] \cdot \bar{H}' \cdot \bar{H}^* da}{2Z_o I_o^2 z} \quad (135)$$



Evaluation of the numerator of (135) leads to:

$$\Gamma' + \Gamma^* = j\omega\mu_0 \frac{\int_{-\infty}^{+\infty} \int_0^D (\chi_{xx} H_x H_x^* + \chi_{zz} H_z H_z^*) dy dx}{2Z_0 I_{0z}^2} \quad (136)$$

where unperturbed fields have been used as a first approximation to perturbed fields and where because of the geometry no demagnetization of r.f. fields takes place. No cross terms involving products of  $H_z$  and  $H_x$  appear in the numerator of (136) because this product is odd and

$$\int_{-\infty}^{+\infty} H_z H_x^* dx = \int_{-\infty}^{+\infty} H_x H_z^* dx = 0. \quad (137)$$

Referring back to equations (2) and (5), substituting in equation (136) and applying the Parseval's theorem one obtains:

$$\begin{aligned} \Gamma' + \Gamma^* = \frac{j\omega\mu_0}{2Z_0 I_{0z}^2} \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_0^D \left\{ \chi_{xx} \left[ \pm j\beta(-j\alpha\phi_2^h(\alpha, y) \right. \right. \\ \left. \left. + j\omega\epsilon_2 \frac{\partial\phi_2^e(\alpha, y)}{\partial y} \right] \left[ \mp j\beta(-j\alpha\phi_2^h(\alpha, y) - j\omega\epsilon_2 \frac{\partial\phi_2^e(\alpha, y)}{\partial y}) \right]^* \right. \\ \left. + \chi_{zz} k_{c2}^4 \phi_2^h(\alpha, y) \phi_2^{h*}(\alpha, y) \right\} dy d\alpha \end{aligned} \quad (138)$$

where

$$\chi_{zz} = \chi_{xx}.$$

This propagation constant expression holds for  $\pm z$  directed traveling waves depending upon the choice of upper or lower sign in (138).



One might attempt to solve this equation by defining regions of  $\alpha$  where  $\gamma_2$  has imaginary and real values which are indicated by equations (36) and (37).

Also substituting previously calculated corresponding equations of  $\Phi_2^h(\alpha, y)$  and  $(\partial \Phi_2^e(\alpha, y))/\partial y$  for hyperbolic and trigonometric cases into (138) and integrating out the  $y$  dependence the following is obtained:

$$\Gamma' + \Gamma^* = \frac{1}{16\pi} \cdot \frac{j\omega\mu_0}{Z_0 I_{0z}^2} \int_{\text{TRIG REGION}} \left( \frac{2\gamma_2''^D + \text{Sin}2\gamma_2''^D}{\gamma_2''} \right) \\ \left\{ \chi_{xx} [(-\beta^2 \alpha^2 + k_{c2}^4) |C_T^h(\alpha)|^2 \pm j\omega\epsilon_2 \beta \alpha \gamma_2'' \right. \\ \left. (-jC_T^h(\alpha) B_T^{*e}(\alpha) - jC_T^{h*}(\alpha) B_T^e(\alpha)) - (\omega\epsilon_2 \gamma_2'')^2 |B_T^e(\alpha)|^2 \right. \\ \left. \right\} d\alpha + \frac{1}{16\pi} \frac{j\omega\mu_0}{Z_0 I_{0z}^2} \int_{\text{HYP REGION}} \left( \frac{\text{Sinh}2\gamma_2^D - 2\gamma_2^D}{\gamma_2} \right) \\ \left\{ \chi_{xx} [(-\beta^2 \alpha^2 + k_{c2}^4) |C_H^h(\alpha)|^2 \pm j\omega\epsilon_2 \beta \alpha \gamma_2 (C_H^h(\alpha) B_H^{e*}(\alpha) \right. \\ \left. - C_H^{h*}(\alpha) B_H^e(\alpha)) - (\omega\epsilon_2 \gamma_2)^2 |B_H^e(\alpha)|^2 \right\} d\alpha. \quad (139)$$

Evaluating the difference between perturbed and unperturbed propagation constants,

$$\Gamma' + \Gamma^* = \Delta\alpha + j\Delta\beta \\ \Gamma' + \Gamma^* = \alpha' + j\beta' - j\beta \quad (140)$$

where



$$\Delta\alpha = \alpha' \text{ and } \Delta\beta = \beta' - \beta.$$

Also, as stated in Reference 5

$$\chi_{xx} = \chi'_{xx} - j\chi''_{xx}. \quad (141)$$

Substituting (140) and (141) into equation (139) and separating real and imaginary parts gives expressions for  $\alpha'$  and  $\beta'$ , loss and phase shift with ferrite respectively.

Define a common expression

$$K^{\pm} = \frac{1}{16\pi} \cdot \frac{j\omega\mu_0}{Z_0 I_{0z}^2} \left\{ \int_{\text{TRIG REGION}} \left( \frac{2\gamma_2'' + \sin 2\gamma_2'' D}{\gamma_2''} \right) \right. \\ \left[ (-\beta^2 \alpha^2 + k_{c2}^4) |C_T^h(\alpha)|^2 \pm 2\omega\epsilon_2 \beta \alpha \gamma_2'' (C_T^h(\alpha) B_T^{e*}(\alpha)) \right. \\ \left. - (\omega\epsilon_2 \gamma_2'')^2 |B_T^e(\alpha)|^2 \right] d\alpha + \int_{\text{HYP REGION}} \left( \frac{\sinh 2\gamma_2 D - 2\gamma_2 D}{\gamma_2} \right) \\ \left. \left[ (-\beta^2 \alpha^2 + k_{c2}^4) |C_H^h(\alpha)|^2 \pm 2\omega\epsilon_2 \beta \alpha \gamma_2 (-j C_H^{*h}(\alpha) B_H^e(\alpha)) \right. \right. \\ \left. \left. - (\omega\epsilon_2 \gamma_2)^2 |B_H^e(\alpha)|^2 \right] d\alpha \right\} \quad (142)$$

Then

$$\Delta\alpha^{\pm} = K^{\pm} \chi''_{xx} \quad (143)$$

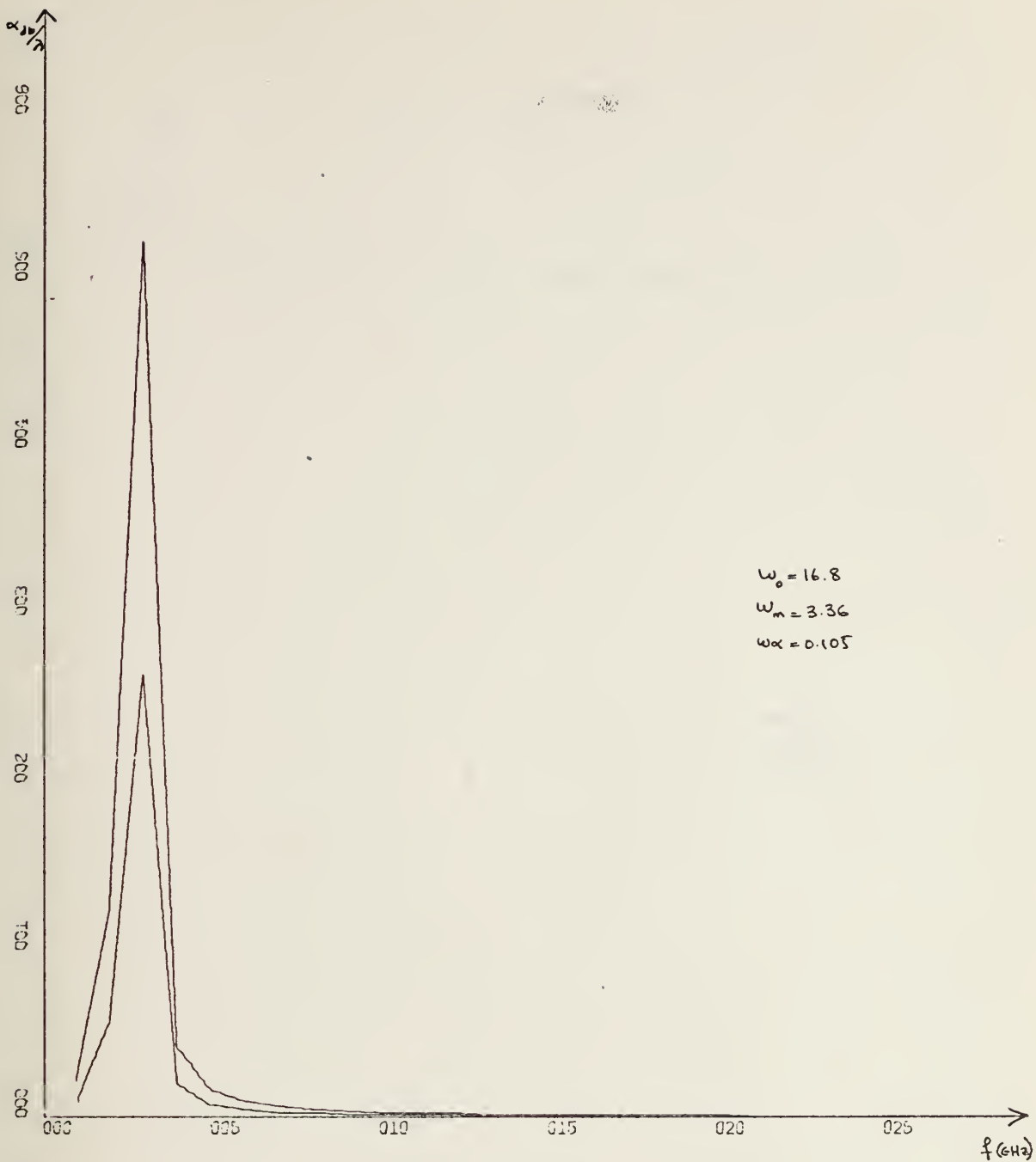
$$\Delta\beta^{\pm} = K^{\pm} \chi'_{xx} \quad (144)$$

where  $\chi'_{xx}$  and  $\chi''_{xx}$  are defined in Reference 5.



Loss in db per wavelength versus frequency and  $\beta_f/\beta$  versus frequency are plotted in Figures 10 and 11.

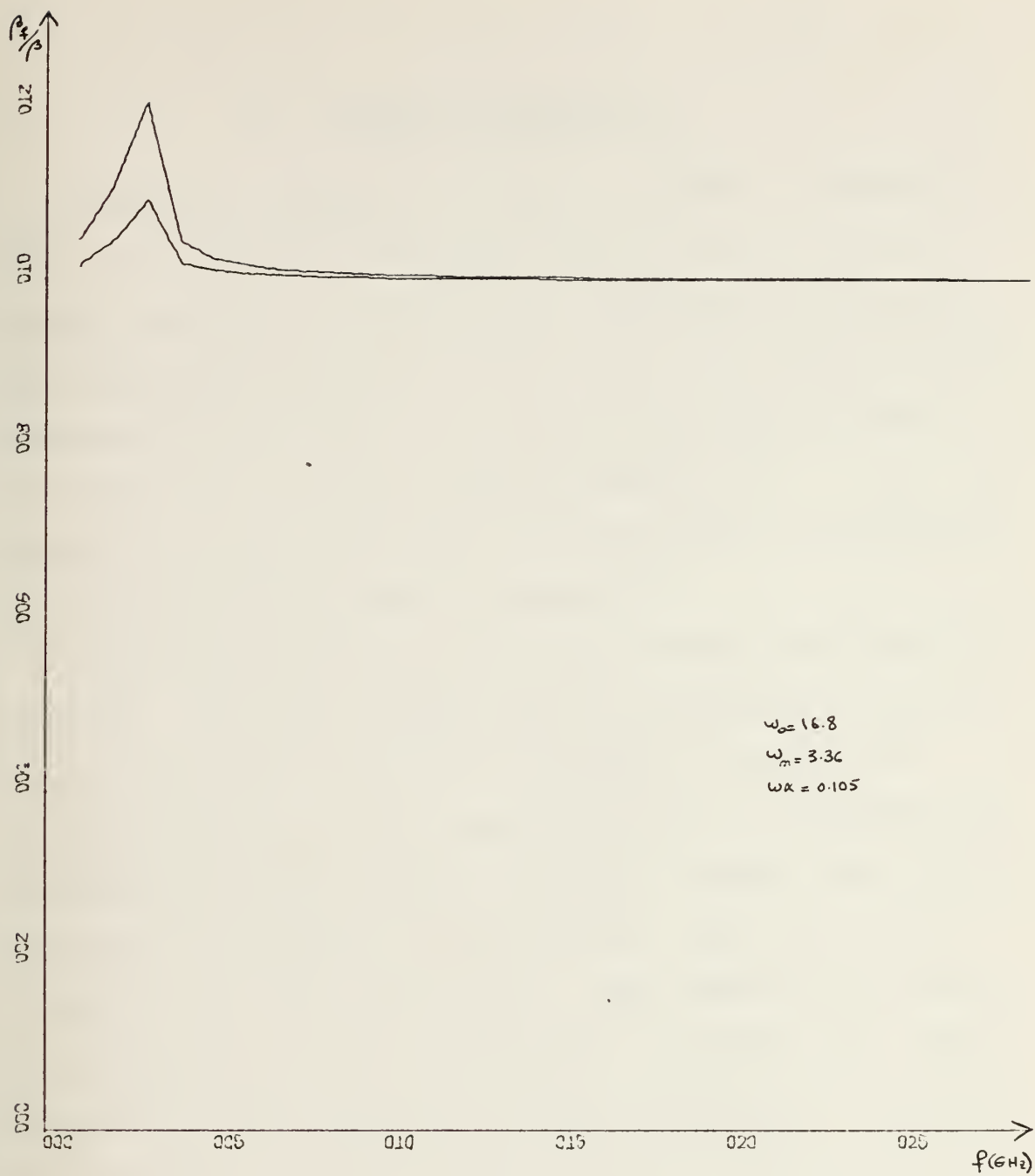




X-SCALE=5.00E+00 UNITS INCH.  
 Y-SCALE=1.00E-01 UNITS INCH.

Figure 10. Loss per wavelength in db versus frequency (GHz).





X-SCALE=5.00E+00 UNITS INCH.  
 Y-SCALE=2.00E-01 UNITS INCH.

Figure 11.  $\beta_f/\beta$  versus frequency (GHz).



#### IV. COMPUTER PROGRAMMING

A computer program is written in FORTRAN IV language that first calculates the phase velocity or  $\lambda/\lambda'$  ratio for a given geometry of microstrip and parameters of substrate. Then the program proceeds and calculates the characteristic impedance for the same set of data using results found in the first phase of the calculation namely  $\lambda/\lambda'$ . If the substrate is ferrite, also phase shift and loss per wavelength are calculated and curves are generated.

The program consists of one main routine and three sub-routines. The main routine starts by calculating all constants and parameters which are needed to evaluate the various integrals. First equation (105) is integrated and evaluated for an arbitrary initial  $\lambda/\lambda'$  value. This process continues for different values of  $\lambda/\lambda'$  until zero is obtained as the value of integral. This computer time consuming iteration process is shortened by a simple root finding routine that determines the correct region of  $\lambda/\lambda'$  and eliminates others. On the average, after 9 iterations equation (105) gives a value which will yield zero value for the integral with an accuracy of 0.01%. The exact number of steps will change depending on desired accuracy and physical parameters.

Evaluation of the integral is done by standard IBM 360/67 system library subroutine DQG24 which uses a 24



point Gaussian Quadrature formula which integrates polynomials up to 47th degree.

This routine has been found to accurately integrate the type of curves encountered in this study.

The DQG24 routine requires an external function subprogram which defines the polynomial to be integrated for imaginary and real values of  $\gamma_2$ . Two different subprograms GZR, GZIM were used for this purpose and to test and set the limits of integration, for real and imaginary character of  $\gamma_2$  respectively.

It is also proven that increasing the limits of integration or using a 32 point integration routine which evaluates polynomials up to 64th degree does not change the result more than 0.1%.

Another subroutine called TRAN supplies the expression for the transformed current distribution depending upon the physical configuration of the microstrip.

As far as the theoretical development and programming is concerned the only approximations used are the representation of the axial surface current density by a square pulse and the assumption that the transverse surface current density is zero. Different forms of current distributions were tried such as

$$J_z(x) = \begin{cases} \cosh x & \text{on the strip} \\ 0 & \text{elsewhere.} \end{cases}$$



It was concluded that due to the many zero crossings of this transformed current distribution curve and the greater number of side lobes which adversely affected programming efficiency its use was not justified even though it more accurately approximates the true current distribution.

Characteristic impedance was calculated as a next step by evaluating equations (129), (130) and (132) using different parts of the same routines used in the first part of the calculation.

If the substrate is ferrite the program proceeds to another section which evaluates equations (142) through (144) and calculates loss and phase shift values as a function of operation frequency.

It may be pertinent to quote the typical computational time using the method outlined in this study.

From the GO step the average time for calculating  $\lambda/\lambda'$ ,  $Z_0$ , and loss and phase shift (if necessary) for a single frequency was between 1.5 to 1.9 sec on an IBM 360/67.

The computer program accepts the following data:

- Substrate parameters,  $\epsilon_{r_2}$ ,  $\mu_{r_2}$ .
- Thickness of substrate,  $D$ , in millimeters.
- The separation between conductors,  $S$ , in millimeters (only for coupled strip).
- Initial frequency (GHz).

The program starts calculating values for the initial frequency and repeats the procedure by increasing the frequency in 1 GHz steps up to 50 times. Means are also provided to compare the half wavelength,  $\lambda/2$ , with the thickness



of the substrate, D. For

$$D = \frac{\lambda}{2}$$

the program stops due to the presence of higher order modes.



## V. NUMERICAL RESULTS AND COMPARISONS

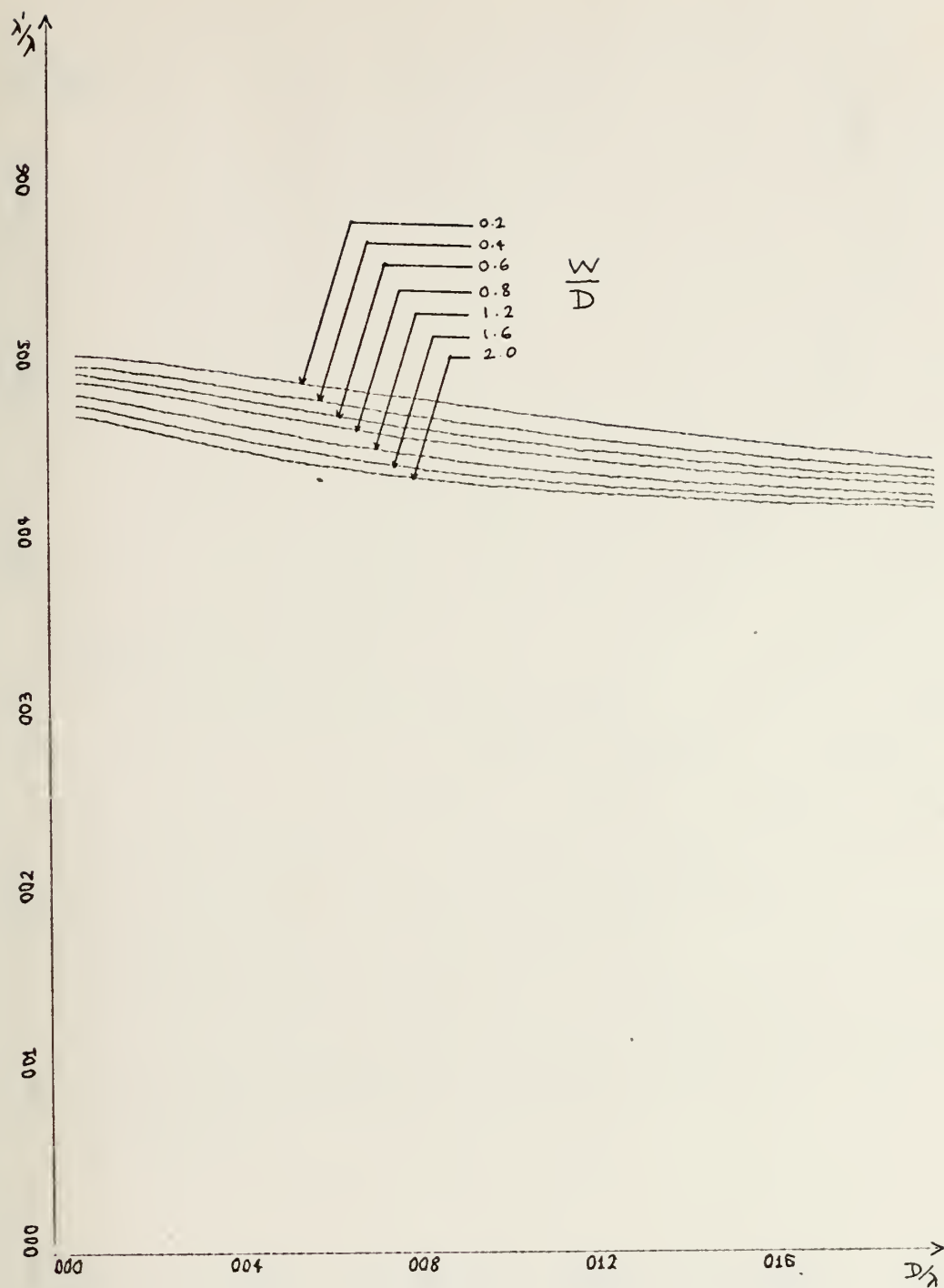
The dispersion characteristic of the microstrip as a function of frequency has been evaluated for a variety of geometries and these results are presented both in tabular and plotted form. In addition, characteristic impedance as a function of frequency is presented for the same geometries and dielectric constant.

Figures 12 through 15 show the frequency variation of wavelength and characteristic impedance of a single strip for various values of  $W/D$ . Similar families of curves for coupled strips appear in Figures 16 through 19. In this case the parameter is  $S/D$  and  $W/D$  is held constant.

The results indicate that higher frequencies and wider strips increase the effective dielectric constant,  $\epsilon_{r_{eff}}$ , and the characteristic impedance  $Z_0$  of both single and coupled strips. The spacing  $S$  between coupled strips also has an effect on  $\epsilon_{r_{eff}}$ . Increasing spacing,  $S$ , increases  $\epsilon_{r_{eff}}$  for odd and even modes. The effect on characteristic impedance depends upon the mode of propagation. Increasing spacing,  $S$ , increases  $Z_0$  for the odd coupled mode but causes a decrease in  $Z_0$  for the even coupled mode.

All available published data is based on quasi-static analyses except in a few cases. Basically, in this study a third dimension, frequency or  $D/\lambda$ , is introduced. Two 3-D plots are given in Figures 20 and 21 to illustrate this point. The x-axis is  $D/\lambda$ , the y axis  $W/D$ , and the z-axis



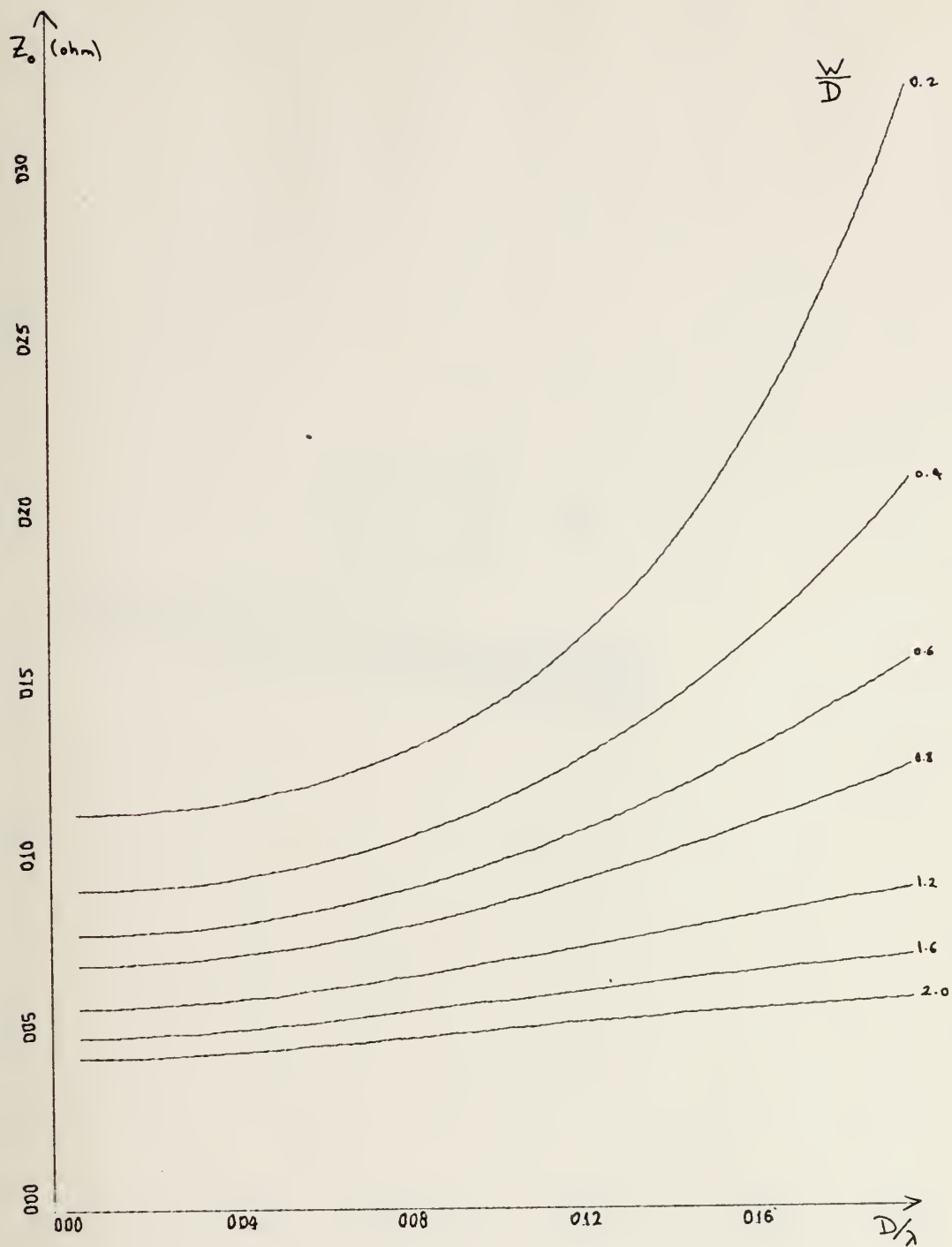


X-SCALE =  $4.00\text{E}-02$  UNITS INCH.

Y-SCALE =  $1.00\text{E}-01$  UNITS INCH.

Figure 12. Single Strip  $\lambda'/\lambda$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 6.0$ .



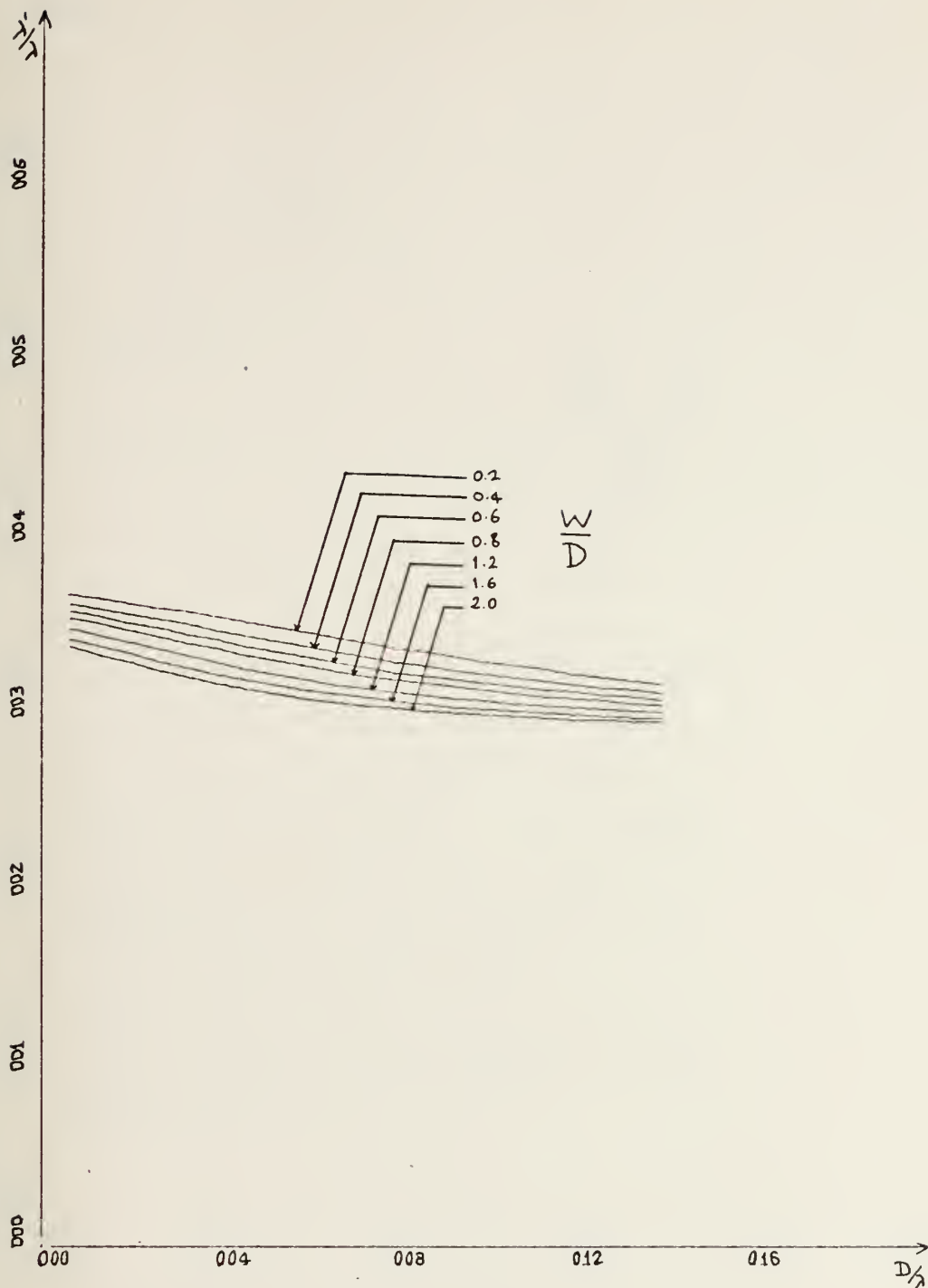


X-SCALE =  $4.00 \times 10^{-2}$  UNITS INCH.

Y-SCALE =  $5.00 \times 10^1$  UNITS INCH.

Figure 13. Single strip  $Z_0$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 6.0$ .



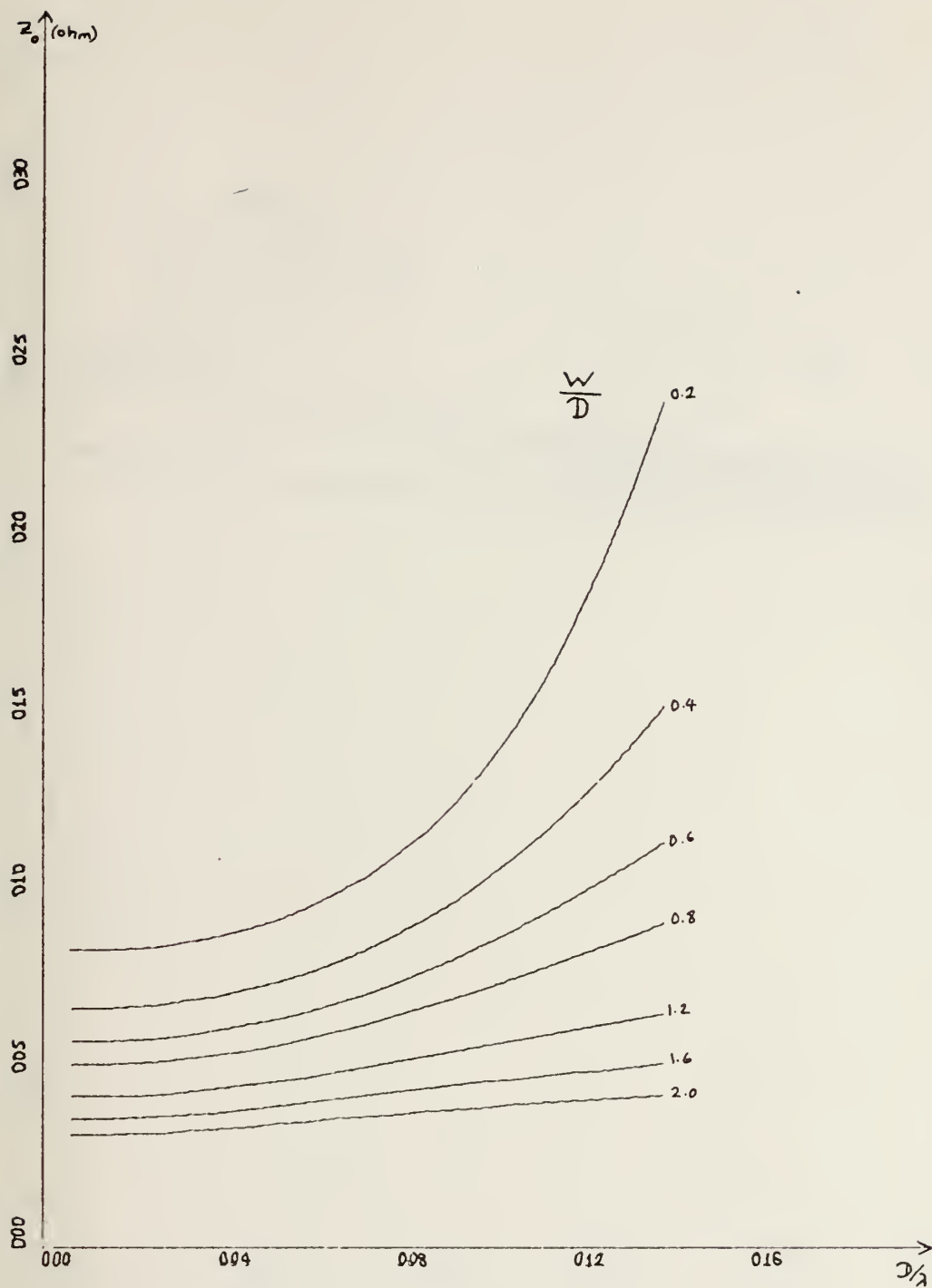


X-SCALE =  $4.00 \times 10^{-2}$  UNITS INCH.

Y-SCALE =  $1.00 \times 10^{-1}$  UNITS INCH.

Figure 14. Single strip  $\lambda'/\lambda$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 12.0$



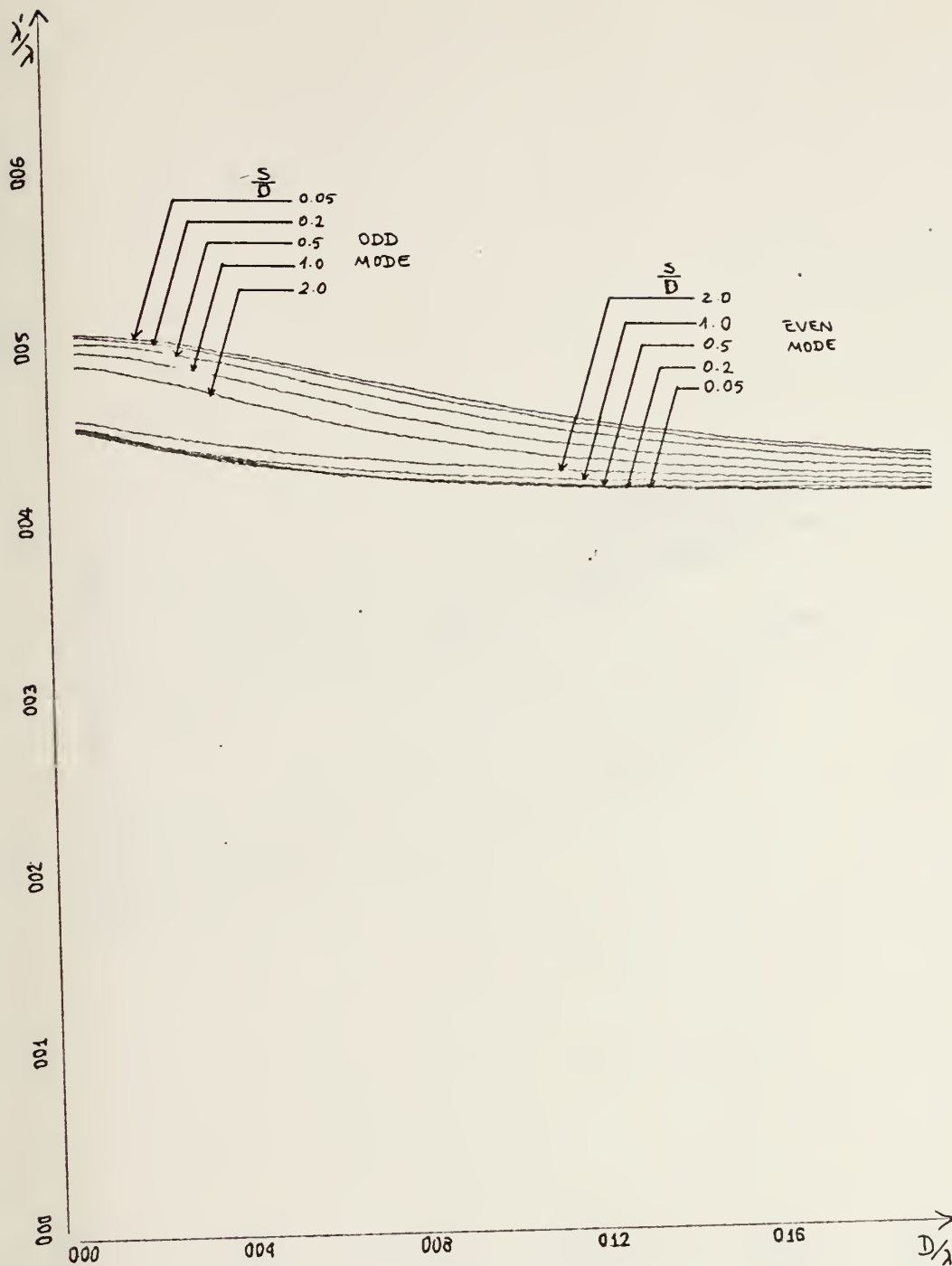


X-SCALE=4.00E-02 UNITS INCH.

Y-SCALE=5.00E+01 UNITS INCH.

Figure 15. Single strip  $Z_0$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 12.0$ .

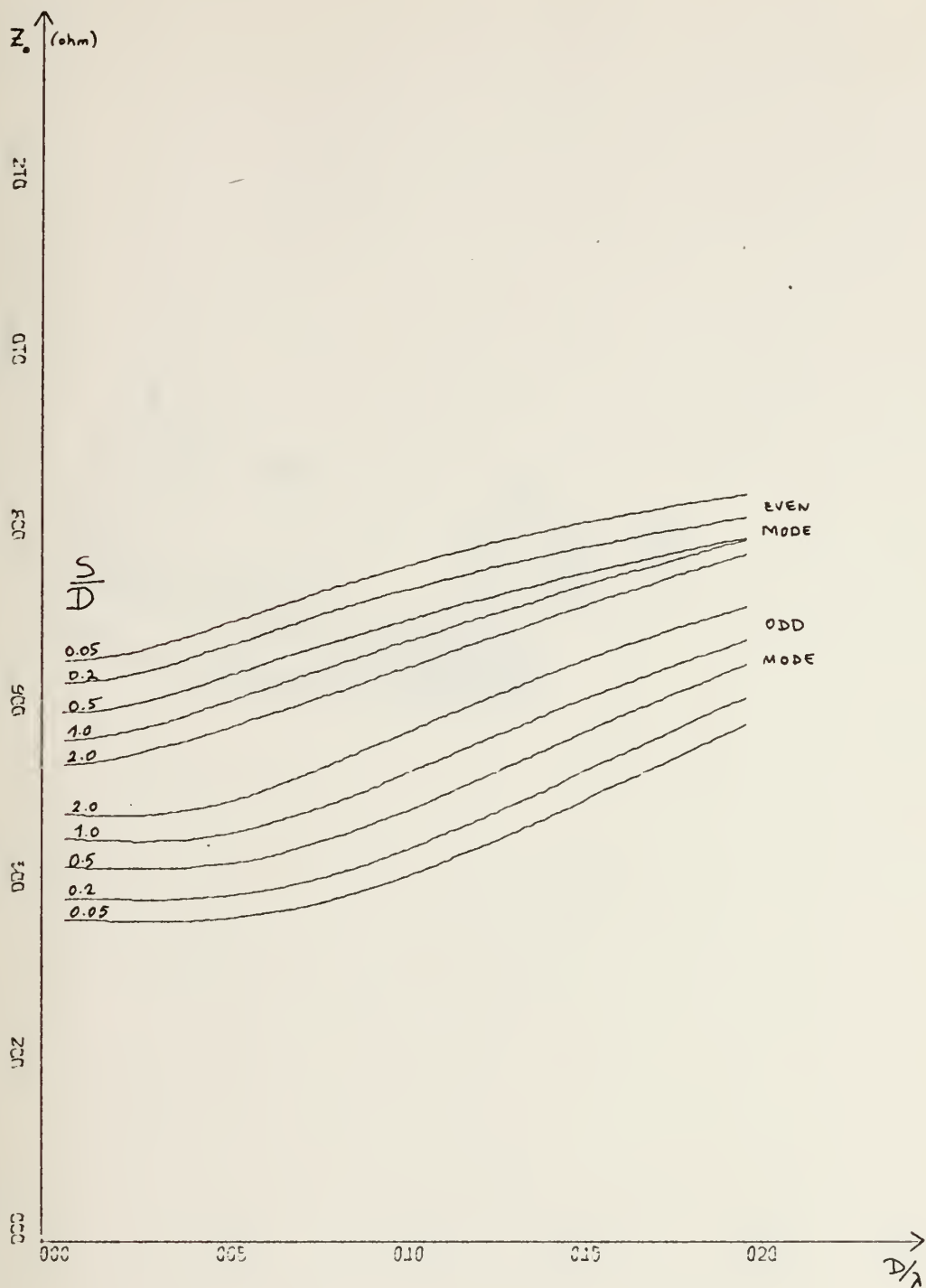




X-SCALE =  $4.00 \times 10^{-2}$  UNITS INCH  
Y-SCALE =  $1.00 \times 10^{-1}$  UNITS INCH

Figure 16. Coupled strip even and odd modes  $\lambda'/\lambda$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 6.0$   $W/D = 1.54$ .



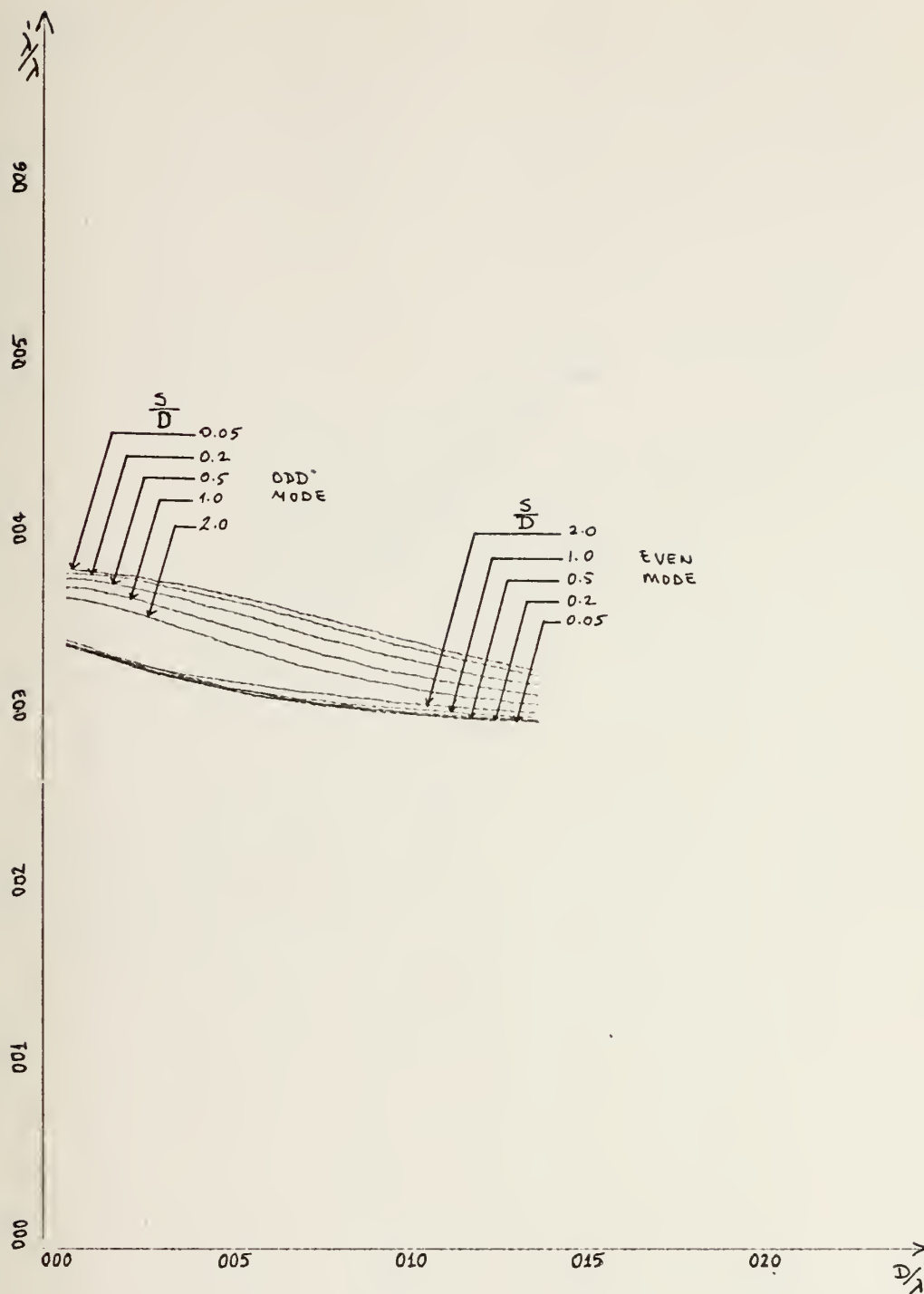


X-SCALE=5.00E-02 UNITS INCH.

Y-SCALE=2.00E+01 UNITS INCH.

Figure 17. Coupled strip even and odd modes  $Z_0$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 6.0$   $W/D = 1.54$ .



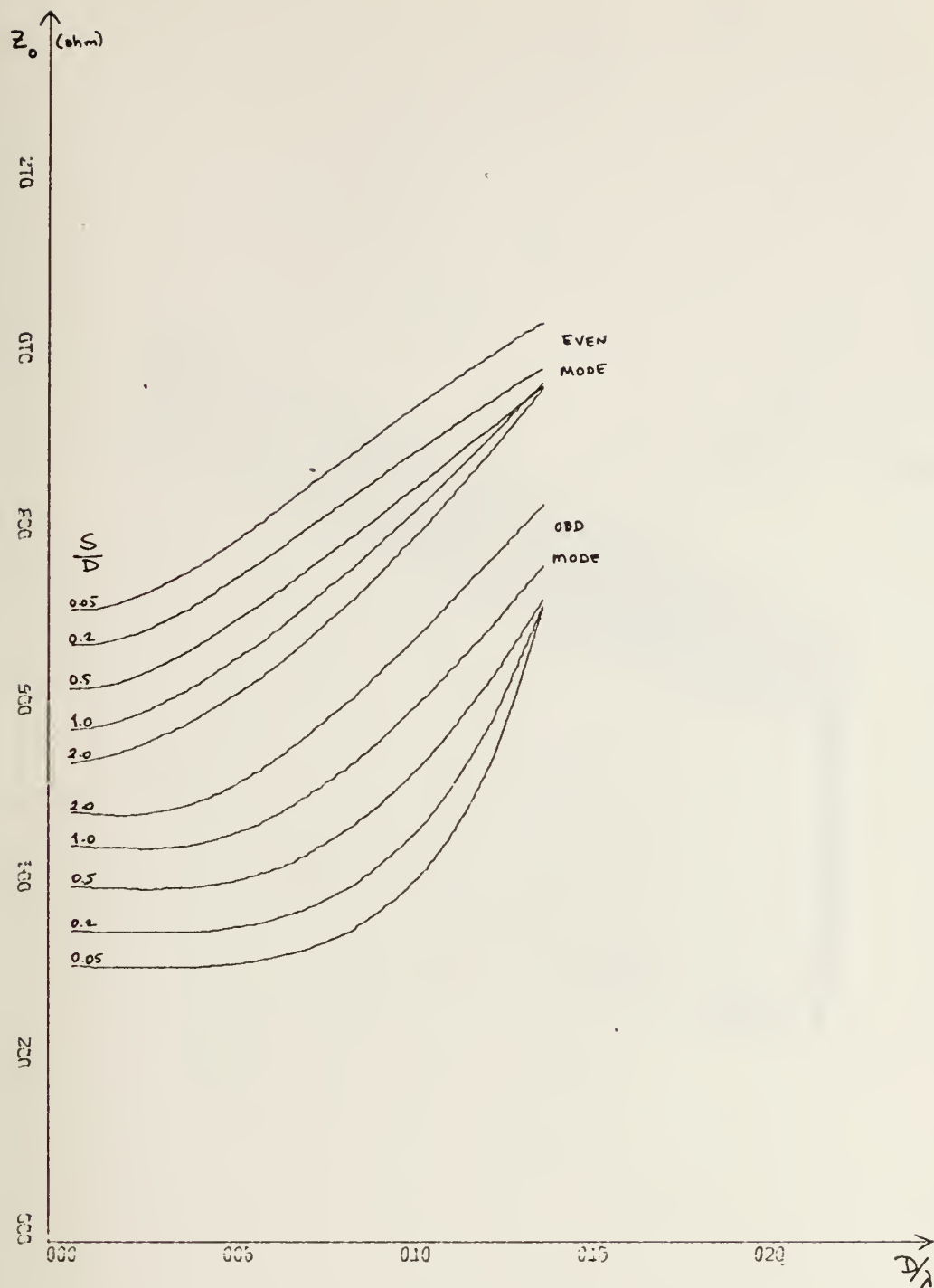


X-SCALE 5.00E-02 UNITS INCH.

Y-SCALE 1.00E-01 UNITS INCH.

Figure 18. Coupled strip even and odd modes  $\lambda'/\lambda$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 12.0$   $W/D = 0.8$ .





X-SCALE: 5.00E-02 UNITS INCH.

Y-SCALE: 2.00E+01 UNITS INCH.

Figure 19. Coupled strip even and odd modes  $Z_0$  versus  $D/\lambda$  curves  $\epsilon_{r2} = 12.0$   $W/D = 0.8$ .



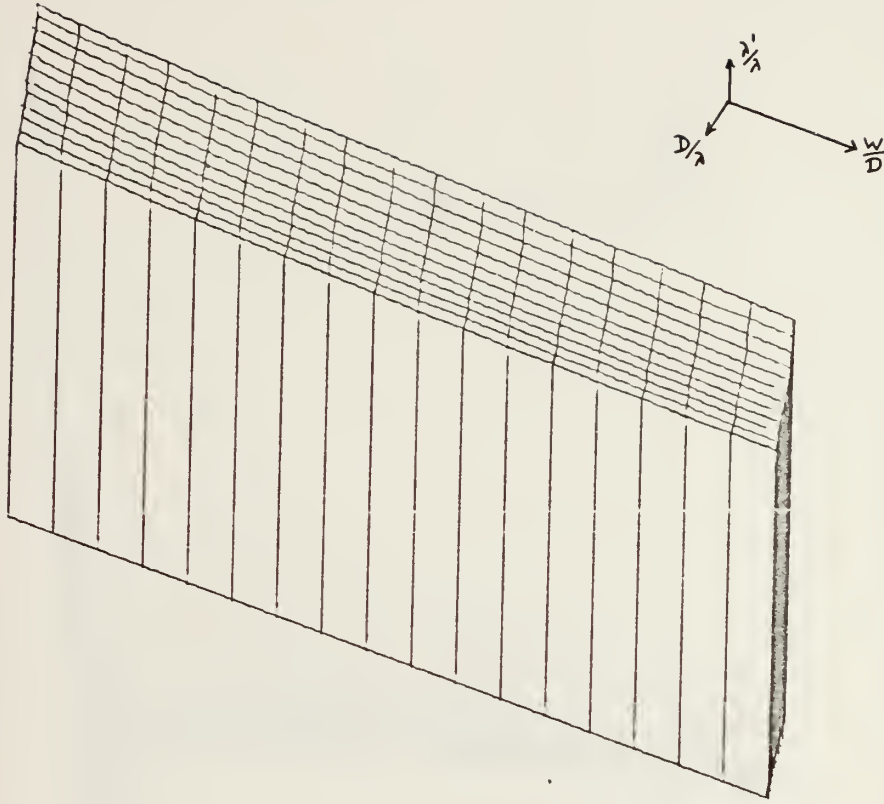


Figure 20. Three dimensional plot  $\lambda'/\lambda$ ,  $W/D$ ,  $D/\lambda$  single strip.



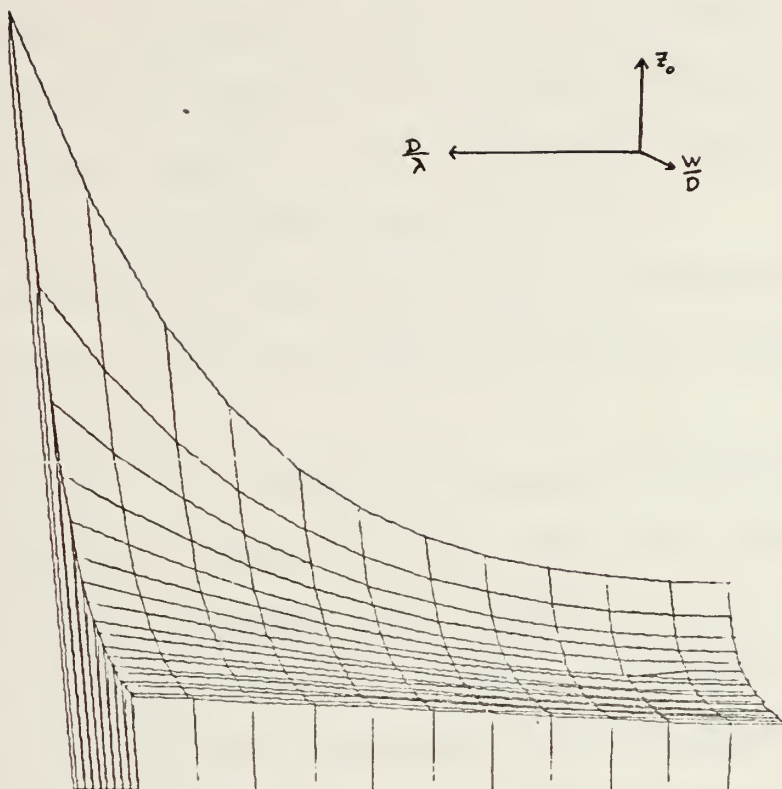


Figure 21. Three dimensional plot  $Z_0$ ,  $W/D$ ,  $D/\lambda$  single strip.



$\lambda'/\lambda$  and  $Z_0$  respectively. One can find the  $\lambda'/\lambda$  versus  $W/D$  and  $Z_0$  versus  $W/D$  surfaces already plotted in different references, i.e. [Ref. 10] but only for zero frequency.

This newly added dimension makes it possible to find data for a specific frequency in addition to a certain geometry.

In Table I the values found for characteristic impedance with two different methods in Reference 3 and generated from this study are compared. This comparison reflects the agreement in the low frequency range.

Also some of the tabulated data in Reference 11 is compared with data obtained from this study in Table II for coupled strips.

One of the few studies of frequency dependence was by Haddad [Ref. 9]. Table III shows data taken from Haddad's work and from this study which clearly indicates the agreement also for low frequency ranges. Although Haddad's analysis was frequency dependent, the data presented by him was restricted to relatively low frequencies.

Finally three sample computer output pages are given in Tables IV, V and VI.

Although the wavelength and characteristic impedance of microstrips on dielectric substrate depend only upon  $D/\lambda$ , the normalized frequency, this is not true for ferrite substrates. Since the computer program will calculate results for either case it is necessary to provide actual dimensions,  $D$ ,  $W$ ,  $S$  etc. as inputs. Where both frequency and  $D/\lambda$  are



TABLE I. Characteristic Impedances of Microstrip Transmission Lines Single Strip.

$$\epsilon_{r_2} = 6.0$$

<u>W/D</u>	<u><math>Z_o^*</math> (ohm)</u>	<u><math>Z_o^\dagger</math> (ohm)</u>	<u><math>Z_o^s</math> (ohm)</u>
0.1	135.455	134.352	137.64
0.2	113.272	112.255	115.41
0.4	91.172	89.909	93.25
0.7	73.613	71.995	73.53
1.0	62.713	60.970	64.45
2.0	43.149	41.510	44.15
4.0	27.301	26.027	27.23
10.0	13.341	12.485	12.58

$$\epsilon_{r_2} = 16.0$$

0.1	85.9659	87.762	87.70
0.2	71.6954	73.025	73.43
0.4	57.4999	58.110	59.20
0.7	46.2344	46.217	47.83
1.0	39.2512	38.948	40.72
2.0	26.7555	26.248	27.73
4.0	16.7210	16.300	16.96
10.0	8.0385	7.807	7.77

\* Characteristic impedance obtained by method of moment [Ref. 3]

† Characteristic impedance obtained by conformal mapping [Ref. 3]

s Characteristic impedance obtained by spectral domain method in this study.



Table II. Effective Dielectric Constant and Characteristic Impedance for Coupled Strips.

W/H	EVEN MODE		ODD MODE		$Z_O^*$ (ohm)	$Z_O^\dagger$ (ohm)
	$\epsilon_{\text{eff}}^*$	$\epsilon_{\text{eff}}^\dagger$	$\epsilon_{\text{eff}}^*$	$\epsilon_{\text{eff}}^\dagger$		
0.1	6.25	6.25	5.50	5.51	64.92	65.29
0.3	6.54	6.54	5.52	5.54	45.97	49.02
0.5	6.77	6.76	5.56	5.59	39.15	43.43
0.7	6.96	6.95	5.61	5.66	35.09	39.95
0.9	7.13	7.12	5.67	5.73	32.19	37.29
1.1	7.28	7.28	5.73	5.82	29.92	35.06
1.3	7.41	7.42	5.79	5.90	28.05	33.12
1.5	7.53	7.55	5.85	6.00	26.46	31.83
1.7	7.63	7.67	5.92	6.09	25.07	29.80
1.9	7.73	7.77	5.99	6.19	23.85	28.36

## \* Weiss Results

† Spectral Domain Transform method used in this study.



Table III. Frequency Dependence of Single and Coupled Microstrip.

SINGLE W/D = 1.0		COUPLED (EVEN MODE) W/D = 1.0 S/D = 0.4		COUPLED (ODD MODE) W/D = 1.0 S/D = 0.4	
$\epsilon_{r\text{eff}}^*$	$\epsilon_{r\text{eff}}^\dagger$	$\epsilon_{r\text{eff}}^*$	$\epsilon_{r\text{eff}}^\dagger$	$\epsilon_{r\text{eff}}^*$	$\epsilon_{r\text{eff}}^\dagger$
1GHZ 6.78	6.78	7.39	7.43	5.84	5.85
2GHZ 7.05	6.98	7.8	7.72	5.87	5.94
$Z_o^*$	$Z_o^\dagger$	$Z_o^*$	$Z_o^\dagger$	$Z_o^*$	$Z_o^\dagger$
1GHZ 48.5 $\Omega$	50.88 $\Omega$	61.8 $\Omega$	62.24 $\Omega$	36.2 $\Omega$	39.31 $\Omega$
2GHZ 50.0 $\Omega$	51.22 $\Omega$	63.0 $\Omega$	62.8 $\Omega$	36.2 $\Omega$	39.18 $\Omega$

\* Haddad results

† Spectral Domain Transform method used in this study.



Table IV. Sample Computer Output for Single Strip.

W = 1mm

D = 2mm

$\epsilon_{r2} = 12$

W/D = 0.5

FREQUENCY	D/LAMBDA	L/LPR	LPR/L	CH IMPEDANCE
1.000 GHZ	0.00667	2.75971	0.36236	62.59 OHM
2.000 GHZ	0.01333	2.78140	0.35953	62.70 OHM
3.000 GHZ	0.02000	2.80726	0.35622	63.07 OHM
4.000 GHZ	0.02667	2.83508	0.35272	63.72 OHM
5.000 GHZ	0.03333	2.86373	0.34919	64.68 OHM
6.000 GHZ	0.04000	2.89254	0.34572	65.96 OHM
7.000 GHZ	0.04667	2.92108	0.34234	67.56 OHM
8.000 GHZ	0.05333	2.94907	0.33909	69.51 OHM
9.000 GHZ	0.06000	2.97628	0.33599	71.81 OHM
10.000 GHZ	0.06667	3.00258	0.33305	74.47 OHM
11.000 GHZ	0.07333	3.02787	0.33027	77.51 OHM
12.000 GHZ	0.08000	3.05206	0.32765	80.94 OHM
13.000 GHZ	0.08667	3.07512	0.32519	84.77 OHM
14.000 GHZ	0.09333	3.09702	0.32289	89.01 OHM
15.000 GHZ	0.10000	3.11775	0.32074	93.66 OHM
16.000 GHZ	0.10667	3.13733	0.31874	98.73 OHM
17.000 GHZ	0.11333	3.15577	0.31688	104.24 OHM
18.000 GHZ	0.12000	3.17311	0.31515	110.17 OHM
19.000 GHZ	0.12667	3.18939	0.31354	116.52 OHM
20.000 GHZ	0.13333	3.20464	0.31205	123.31 OHM



Table V. Sample Computer Output for Coupled Strip (Odd Mode)

W = 1.6mm D = 2.0mm S = 0.8mm W/D = 0.8 S/D = 0.4  $\epsilon_{r_2} = 12$

FREQUENCY	D/LAMBDA	L/LPR	LPR/L	CH IMPEDANCE
1.000 GHZ	0.00667	2.61034	0.38309	39.05 OHM
2.000 GHZ	0.01333	2.61858	0.38189	38.97 OHM
3.000 GHZ	0.02000	2.63141	0.38002	38.89 OHM
4.000 GHZ	0.02667	2.64815	0.37762	38.84 OHM
5.000 GHZ	0.03333	2.66821	0.37478	38.87 OHM
6.000 GHZ	0.04000	2.69112	0.37159	39.00 OHM
7.000 GHZ	0.04667	2.71640	0.36813	39.26 OHM
8.000 GHZ	0.05333	2.74366	0.36446	39.69 OHM
9.000 GHZ	0.06000	2.77249	0.36069	40.31 OHM
10.000 GHZ	0.06667	2.80248	0.35683	41.16 OHM
11.000 GHZ	0.07333	2.83325	0.35295	42.26 OHM
12.000 GHZ	0.08000	2.86441	0.34911	43.65 OHM
13.000 GHZ	0.08667	2.89561	0.34535	45.34 OHM
14.000 GHZ	0.09333	2.92649	0.34171	47.37 OHM
15.000 GHZ	0.10000	2.95675	0.33821	49.75 OHM
16.000 GHZ	0.10667	2.98613	0.33488	52.51 OHM
17.000 GHZ	0.11333	3.01442	0.33174	55.67 OHM
18.000 GHZ	0.12000	3.04146	0.32879	59.23 OHM
19.000 GHZ	0.12667	3.06714	0.32604	63.22 OHM
20.000 GHZ	0.13333	3.09139	0.32348	67.65 OHM



Table VI. Sample Computer Output for Coupled Strip (Even Mode)

W=1.6mm D=2.0mm S=0.8mm W/D = 0.8 S/D = 0.4  $\epsilon_{r2} = 12$

FREQUENCY	D/LAMBDA	L/LPR	LPR/L	CH IMPEDANCE
1.000 GHZ	0.00667	2.92702	0.34164	64.22 OHM
2.000 GHZ	0.01333	2.96430	0.33735	64.38 OHM
3.000 GHZ	0.02000	3.00492	0.33279	64.92 OHM
4.000 GHZ	0.02667	3.04470	0.32844	65.82 OHM
5.000 GHZ	0.03333	3.08186	0.32448	67.00 OHM
6.000 GHZ	0.04000	3.11570	0.32096	68.42 OHM
7.000 GHZ	0.04667	3.14609	0.31786	70.00 OHM
8.000 GHZ	0.05333	3.17316	0.31514	71.70 OHM
9.000 GHZ	0.06000	3.19719	0.31278	73.49 OHM
10.000 GHZ	0.06667	3.21849	0.31070	75.35 OHM
11.000 GHZ	0.07333	3.23740	0.30889	77.26 OHM
12.000 GHZ	0.08000	3.25422	0.30729	79.20 OHM
13.000 GHZ	0.08667	3.26921	0.30588	81.16 OHM
14.000 GHZ	0.09333	3.28263	0.30463	83.14 OHM
15.000 GHZ	0.10000	3.29467	0.30352	85.13 OHM
16.000 GHZ	0.10667	3.30552	0.30252	87.13 OHM
17.000 GHZ	0.11333	3.31533	0.30163	89.14 OHM
18.000 GHZ	0.12000	3.32423	0.30082	91.15 OHM
19.000 GHZ	0.12667	3.33234	0.30009	93.17 OHM
20.000 GHZ	0.13333	3.33975	0.29942	95.18 OHM



shown as printed output the calculation is based on the substrate thickness,  $D$ , provided as an input. A width of  $D = 2\text{mm}$  has been used in the calculations presented here.



## VI. CONCLUSIONS

In this thesis a hybrid mode analysis of single and coupled microstrips (of equal width) on dielectric substrate was described. The resulting equations were solved by the method of moments applied in the spectral or (Fourier) transform domain. A computer program was developed to calculate wavelength and characteristic impedance versus frequency. The original results obtained by this method were compared in the low frequency limit with the quasi-static results of several other investigators and were found to be in excellent agreement.

The method was extended to microstrip on ferrite where the propagation constant was calculated using perturbation theory. Numerical evaluation of resulting expressions is again carried out in the spectral domain. A computer program has been developed for this analysis also.



## APPENDIX A

### AUXILIARY VECTOR POTENTIAL FUNCTIONS

#### A. MAGNETIC HERTZIAN VECTOR POTENTIAL FUNCTION

Consider a homogeneous, source-free, isotropic region; hence, there is no charge density and

$$\bar{\nabla} \cdot \bar{E} = 0 \quad (A1)$$

From Vector Algebra, it is known that the divergence of a vector is zero if the vector is, in turn, the curl of another vector; therefore, one can state,

$$\bar{\nabla} \cdot \bar{\nabla} \times \bar{\pi}_h = 0 \quad \nabla \times \bar{\pi}_h \quad (A2)$$

$$\bar{E} = -j\omega\mu \bar{\nabla} \times \bar{\pi}_h. \quad (A3)$$

For time-varying fields

$$\bar{\nabla} \times \bar{H} = j\omega\epsilon \bar{E} \quad (A4)$$

so applying equation (A3),

$$\begin{aligned} \bar{\nabla} \times \bar{H} &= j\omega\epsilon (-j\omega\mu \bar{\nabla} \times \bar{\pi}_h) \\ &= k^2 \bar{\nabla} \times \bar{\pi}_h \\ &= k^2 \bar{\nabla} \times (\bar{\pi}_h + k^{-2} \bar{\nabla} \phi) \end{aligned} \quad (A5)$$

$$\bar{H} = k^2 \bar{\pi}_h + \bar{\nabla} \phi \quad (A6)$$

where,

$$k^2 = \omega^2 \mu \epsilon.$$

Similarly, for time-varying fields,

$$\bar{\nabla} \times \bar{E} = -j\omega\mu \bar{H} \quad (A7)$$



so, applying equation (A3),

$$\bar{\nabla}x(-j\omega\mu\bar{\nabla}x\bar{\pi}_h) = -j\omega\mu(k^2\bar{\pi}_h + \bar{\nabla}\phi) \quad (A8)$$

$$\bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_h - \nabla^2\bar{\pi}_h = k^2\bar{\pi}_h + \bar{\nabla}\phi. \quad (A9)$$

Up to this point  $\phi$  has been defined; arbitrarily choose

$$\phi = \bar{\nabla}\cdot\bar{\pi}_h. \quad (A10)$$

Then, equation (A9) becomes,

$$\nabla^2\bar{\pi}_h + k^2\bar{\pi}_h = 0. \quad (A11)$$

Also, it is known that

$$\bar{\nabla}\cdot\bar{B} = \mu\bar{\nabla}\cdot\bar{H} = 0$$

$$\mu\bar{\nabla}\cdot(k^2\bar{\pi}_h + \bar{\nabla}\phi) = 0 \quad (A12)$$

$$k^2\bar{\nabla}\cdot\bar{\pi}_h + \bar{\nabla}\cdot\bar{\nabla}\phi = 0.$$

Summarizing, for TM modes,

$$\bar{E} = -j\omega\mu\bar{\nabla}x\bar{\pi}_h \quad (A13)$$

$$\begin{aligned} \bar{H} &= k^2\bar{\pi}_h + \bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_h \\ &= \bar{\nabla}x\bar{\nabla}x\bar{\pi}_h. \end{aligned} \quad (A14)$$

Similarly, for TE modes,

$$\bar{H} = j\omega\epsilon\bar{\nabla}x\bar{\pi}_h \quad (A15)$$

$$\begin{aligned} \bar{E} &= k^2\bar{\pi}_e + \bar{\nabla}\bar{\nabla}\cdot\bar{\pi}_e \\ &= \bar{\nabla}x\bar{\nabla}x\bar{\pi}_e. \end{aligned} \quad (A16)$$

## B. TE AND TM MODES FROM VECTOR POTENTIALS

### 1. TE Modes

$$\bar{E} = -j\omega\mu\bar{\nabla}x\bar{\pi}^h \quad (A17)$$



$$\begin{aligned} \bar{H} &= k_c^2 \bar{\pi}^h + \bar{\nabla} \bar{\nabla} \cdot \bar{\pi}^h \\ &= \bar{\nabla}_x \bar{\nabla}_x \bar{\pi}^h \end{aligned} \quad (A18)$$

where  $\bar{\pi}^h$  satisfies:

$$\nabla^2 \bar{\pi}^h + k_c^2 \bar{\pi}^h = 0 \quad (A19)$$

For TE modes,  $E_z = 0$ ; therefore

$$E_z = -j\omega\mu \left( \frac{\partial \pi_y^h}{\partial x} - \frac{\partial \pi_x^h}{\partial y} \right) \quad (A20)$$

from where,

$$\pi_x^h = \pi_y^h = 0. \quad (A21)$$

Let:

$$\bar{\pi}^h = \phi^h e^{-\gamma z} \bar{a}_z \quad (A22)$$

Then, from equation (A13)

$$\nabla_t^2 \phi^h + k_c^2 \phi^h = 0 \quad (A23)$$

where,

$$k_c^2 = \gamma^2 + k^2. \quad (A24)$$

Summarizing,

$$H_z = k_c^2 \phi^h(x, y) e^{\pm \gamma z} \quad (A25)$$

$$\bar{H}_t = \pm \gamma e^{\pm \gamma z} \nabla_t \phi^h \quad (A26)$$

$$\bar{E}_t = \pm \frac{j\omega\mu}{\gamma} \bar{a}_z \times \bar{H}_t \quad (A27)$$

## 2. TM Modes

$$\bar{E} = k_c^2 \bar{\pi}^e + \bar{\nabla} \bar{\nabla} \cdot \bar{\pi}^e \quad (A28)$$

$$= \bar{\nabla}_x \bar{\nabla}_x \bar{\pi}^e$$

$$\bar{H} = j\omega\epsilon \bar{\nabla}_x \bar{\pi}^e \quad (A29)$$



For TM Modes,  $H_z = 0$ ; therefore

$$\overline{\pi}^e = \phi^e(x, y) e^{-\gamma z} \overline{a}_z \quad (\text{A30})$$

and,

$$\nabla_t^2 \phi^e + k_c^2 \phi^e = 0 \quad (\text{A31})$$

where,

$$k_c^2 = \gamma^2 + k^2. \quad (\text{A32})$$

Summarizing,

$$E_z = k_c^2 \phi^e e^{\pm \gamma z} \quad (\text{A33})$$

$$\overline{E}_t = \pm \gamma \overline{V}_t \phi^e e^{\pm \gamma z} \quad (\text{A34})$$

$$\overline{H}_t = \mp \frac{j\omega\epsilon}{\gamma} \overline{a}_z \times \overline{E}_t. \quad (\text{A35})$$

Equations (1) and (2) correspond to equations (A33) and (A25), respectively.



## APPENDIX B

### TRANSVERSE ELECTRIC AND MAGNETIC FIELDS

Consider Maxwell's point form of Amere's law,

$$\nabla \times \vec{H} = j\omega\epsilon \vec{E} \quad (B1)$$

$$\begin{aligned} \bar{a}_x \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) + \bar{a}_y \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) + \bar{a}_z \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \\ = j\omega\epsilon (E_x \bar{a}_x + E_y \bar{a}_y + E_z \bar{a}_z) \end{aligned} \quad (B2)$$

from where,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = j\omega\epsilon E_x \quad (B3)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad (B4)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad (B5)$$

and, due to the  $z$ -dependence of the fields, i.e.,  $e^{\gamma z}$ , one can apply

$$\frac{\partial}{\partial z} e^{\gamma z} = \gamma e^{\gamma z} \quad (B6)$$

obtaining from equations (B3) through (B5)

$$\frac{\partial H_z}{\partial y} - \gamma H_y = j\omega\epsilon E_x \quad (B7)$$

$$\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon E_y \quad (B8)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_z}{\partial y} = j\omega\epsilon E_z. \quad (B9)$$



Similarly, starting from Maxwell's point form of Lenz' law,

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (B10)$$

$$\begin{aligned} \bar{a}_x \left( \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) + \bar{a}_y \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) + \bar{a}_z \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right) \\ = -j\omega\mu (H_x \bar{a}_x + H_y \bar{a}_y + H_z \bar{a}_z) \end{aligned} \quad (B11)$$

from where,

$$\frac{\partial E_z}{\partial y} - \gamma E_y = -j\omega\mu H_x \quad (B12)$$

$$\gamma E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu H_y \quad (B13)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad (B14)$$

Substituting the value of  $E_x$  from equation (B13) into equation (B7), one obtains

$$\begin{aligned} \frac{\partial H_z}{\partial y} - \gamma H_y &= \frac{j\omega\epsilon}{\gamma} \left( \frac{\partial E_z}{\partial x} - j\omega\mu H_y \right) \\ H_y (\gamma^2 + \omega^2\mu\epsilon) &= \gamma \frac{\partial H_z}{\partial y} - j\omega\epsilon \frac{\partial E_z}{\partial x} \end{aligned} \quad (B15)$$

$$H_y = \frac{1}{k_c^2} \left( \gamma \frac{\partial H_z}{\partial y} - j\omega\epsilon \frac{\partial E_z}{\partial x} \right).$$

Similarly, substituting the values of  $E_y$  from equation (B12) into equation (B8), of  $H_x$  from equation (B8) into equation (B12) and of  $H_y$  from equation (B7) into equation (B13), one obtains

- For  $H_x$ ,



$$\gamma H_x - \frac{\partial H_z}{\partial x} = j\omega\epsilon \frac{1}{\gamma} \left( \frac{\partial E_z}{\partial y} + j\omega\mu H_x \right)$$

$$H_x (\gamma^2 + \omega^2 \mu \epsilon) = \gamma \frac{\partial H_z}{\partial x} + j\omega\epsilon \frac{\partial E_z}{\partial y} \quad (B16)$$

$$H_x = \frac{1}{k_c^2} \left( \frac{\partial H_z}{\partial x} + j\omega\epsilon \frac{\partial E_z}{\partial y} \right).$$

- For  $E_y$ ,

$$\frac{\partial E_z}{\partial y} - \gamma E_y = -j\omega\mu \frac{1}{\gamma} \left( \frac{\partial H_z}{\partial x} + j\omega\epsilon E_y \right)$$

$$E_y (\gamma^2 + \omega^2 \mu \epsilon) = \gamma \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x} \quad (B17)$$

$$E_y = \frac{1}{k_c^2} \left( \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x} \right).$$

- For  $E_x$ ,

$$\gamma^2 E_x - \gamma \frac{\partial E_z}{\partial x} = -j\omega\mu \left( \frac{\partial H_z}{\partial y} - j\omega\epsilon E_x \right)$$

$$E_x (\gamma^2 + \omega^2 \mu \epsilon) = \gamma \frac{\partial E_z}{\partial x} - j\omega\mu \frac{\partial H_z}{\partial y} \quad (B18)$$

$$E_x = \frac{1}{k_c^2} \left( \frac{\partial E_z}{\partial x} - j\omega\mu \frac{\partial H_z}{\partial y} \right).$$

Summarizing,

$$E_x = \frac{1}{k_c^2} \left( \gamma \frac{\partial E_z}{\partial x} - j\omega\mu \frac{\partial H_z}{\partial y} \right) \quad (B19)$$

$$E_y = \frac{1}{k_c^2} \left( \gamma \frac{\partial E_z}{\partial y} + j\omega\mu \frac{\partial H_z}{\partial x} \right) \quad (B20)$$

$$H_x = \frac{1}{k_c^2} \left( \gamma \frac{\partial H_z}{\partial x} + j\omega\epsilon \frac{\partial E_z}{\partial y} \right) \quad (B21)$$

$$H_y = \frac{1}{k_c^2} \left( \gamma \frac{\partial H_z}{\partial y} - j\omega\epsilon \frac{\partial E_z}{\partial x} \right). \quad (B22)$$



Equations (B3) through (B6) correspond to equations (B19) through (B22), respectively.



## APPENDIX C

### AVERAGE POWER EXPRESSION IN REGION 2

#### (TRIGONOMETRIC CASE)

Electric and Magnetic field potential functions and derivatives with respect to  $y$  become

$$\Phi_2^e(\alpha, y) = jB_T^e(\alpha) \text{Sin}\gamma_2''y \quad (C1)$$

$$\Phi_2^h(\alpha, y) = C_T^h(\alpha) \text{Cos}\gamma_2''y \quad (C2)$$

$$\frac{\partial \Phi_2^e(\alpha, y)}{\partial y} = j\gamma_2''B_T^e(\alpha) \text{Cos}\gamma_2''y \quad (C3)$$

$$\frac{\partial \Phi_2^h(\alpha, y)}{\partial y} = -\gamma_2''C_T^h(\alpha) \text{Sin}\gamma_2''y. \quad (C4)$$

Substituting into (115)

$$P_{2AVE_T} = \frac{1}{4\pi} \text{Re} \left\{ \int\limits_{\text{TRIG REGION}}^D \int\limits_0^D \left( [-\alpha^2 \beta \omega \epsilon_2 |B_T^e(\alpha) \text{Sin}\gamma_2''y| - \omega \beta \mu_2 \right. \right. \\ \left. \left. |-\gamma_2''C_T^h(\alpha) \text{Sin}\gamma_2''y|^2 - \omega \beta \epsilon_2 |j\gamma_2''B_T^e(\alpha) \text{Cos}\gamma_2''y|^2 - \alpha^2 \omega \mu_2 \beta |C_T^h(\alpha) \text{Cos}\gamma_2''y|^2] \right. \right. \\ \left. \left. + j\alpha k_2^2 [(C_T^h(\alpha) \text{Cos}\gamma_2''y)(-j\gamma_2''B_T^{e*}(\alpha) \text{Cos}\gamma_2''y) + (-jB_T^{e*}(\alpha) \text{Sin}\gamma_2''y) \right. \right. \\ \left. \left. (-\gamma_2''C_T^h(\alpha) \text{Sin}\gamma_2''y)] - j\alpha \beta^2 [(jB_T^e(\alpha) \text{Sin}\gamma_2''y)(-\gamma_2''C_T^{h*}(\alpha) \text{Sin}\gamma_2''y) \right. \right. \\ \left. \left. + (C_T^{h*}(\alpha) \text{Cos}\gamma_2''y)(j\gamma_2''B_T^e(\alpha) \text{Cos}\gamma_2''y)] \right) dy d\alpha \right\} \quad (C5)$$

Then,



$$\begin{aligned}
P_{2AVE_T} = & \frac{1}{4\pi} \operatorname{Re} \left\{ \int\limits_{\substack{\text{TRIG} \\ \text{REGION}}}^D \int_0^D \sin^2 \gamma_2'' y dy [-\alpha^2 \beta \omega \epsilon_2 |B_T^e(\alpha)|^2 \right. \\
& - \omega \beta \mu \gamma_2''^2 |C_T^h(\alpha)|^2 - \alpha^2 k_2^2 \gamma_2'' C_T^h(\alpha) B_T^{e*}(\alpha) - \alpha \beta^2 \gamma_2'' B_T^e(\alpha) C_T^{h*}(\alpha)] d\alpha \\
& + \int\limits_{\substack{\text{TRIG} \\ \text{REGION}}}^D \int_0^D \cos^2 \gamma_2'' y dy [-\omega \beta \epsilon_2 \gamma_2''^2 |B_T^e(\alpha)|^2 - \alpha^2 \omega \mu_2 \beta |C_T^h(\alpha)|^2 \\
& + \alpha k_2^2 \gamma_2'' C_T^h(\alpha) B_T^{e*}(\alpha) + \alpha \beta^2 \gamma_2'' B_T^e(\alpha) C_T^{h*}(\alpha)] d\alpha \left. \right\}. \quad (C6)
\end{aligned}$$

Also the integration with respect to  $y$  in Region 2 may be accomplished analytically as indicated below:

$$\int_0^D \sin^2 \gamma_2'' y dy = \frac{2\gamma_2''^D - \sin 2\gamma_2''^D}{4\gamma_2''} \quad (C7)$$

$$\int_0^D \cos^2 \gamma_2'' y dy = \frac{2\gamma_2''^D + \sin 2\gamma_2''^D}{4\gamma_2''} \quad (C8)$$

By substituting (C7) and (C8) in (C6) and rearranging one can easily obtain

$$\begin{aligned}
P_{2AVE_T} = & -\frac{1}{16\pi} \operatorname{Re} \left\{ \int\limits_{\substack{\text{TRIG} \\ \text{REGION}}} (2\gamma_2''^D - \sin 2\gamma_2''^D) \left[ \frac{\alpha^2 \beta \omega \epsilon_2}{\gamma_2''} |B_T^e(\alpha)|^2 \right. \right. \\
& + \beta \omega \mu_2 \gamma_2'' |C_T^h(\alpha)|^2 + \alpha k_2^2 C_T^h(\alpha) B_T^{e*}(\alpha) + \alpha \beta^2 B_T^e(\alpha) C_T^{h*}(\alpha)] d\alpha + \\
& \int\limits_{\substack{\text{TRIG} \\ \text{REGION}}} (2\gamma_2''^D + \sin 2\gamma_2''^D) \left[ \frac{\alpha^2 \beta \omega \mu_2}{\gamma_2''} |C_T^h(\alpha)|^2 + \beta \omega \epsilon_2 \gamma_2'' |B_T^e(\alpha)|^2 \right. \\
& \left. \left. + \alpha \beta^2 B_T^e(\alpha) C_T^{h*}(\alpha) - k_2^2 C_T^h(\alpha) B_T^{e*}(\alpha) \right] d\alpha \right\}. \quad (C9)
\end{aligned}$$



The final equation (C9) corresponds to equation (130).



# APPENDIX D

THIS PROGRAM IS DEVELOPED TO CALCULATE THE EFFECTIVE WAVELENGTH AND CHARACTERISTIC IMPEDANCE OF SINGLE OR COUPLED MICROSTRIP ON A DIELECTRIC OR FERRITE SUBSTRATE. IF THE SUBSTRATE IS FERRITE LOSS PER WAVELENGTH IN DB AND NORMALIZED PHASE SHIFT ARE ALSO CALCULATED.

PROGRAM ACCEPTS FOLLOWING INPUT DATA

FIRST CARD - IDENTIFICATION DIGITS FORMAT(I3)

THIS THREE DIGIT INTEGER SUPPLIES NECESSARY INFORMATION TO THE PROGRAM

FIRST DIGIT 1 MEANS DIELECTRIC SUBSTRATE

FIRST DIGIT 2 MEANS FERRITE SUBSTRATE

SECOND DIGIT 1 MEANS SINGLE STRIP

SECOND DIGIT 2 MEANS COUPLED STRIP

THIRD DIGIT 1 MEANS EVEN MODE

THIRD DIGIT 2 MEANS ODD MODE

THIRD DIGIT IS USED ONLY FOR COUPLED STRIP WHEN SECOND DIGIT IS EQUAL TO 2. ALL OTHER TIMES SHOULD BE SET TO ZERO.

EXAMPLES

110 SINGLE STRIP DIELECTRIC SUBSTRATE

211 COUPLED STRIP EVEN MODE ON FERRITE SUBSTRATE

SECOND CARD - EPRI, MR1, MR2 FORMAT(3F10.5)

EPRI IS THE RELATIVE DIELECTRIC PERMITTIVITY REGION 1

MR1 IS THE RELATIVE MAGNETIC PERMEABILITY REGION 1

MR2 IS THE RELATIVE MAGNETIC PERMEABILITY REGION 2

THIRD CARD - DD, WW1, SS FORMAT(3F10.5)

DD - THICKNESS OF THE SUBSTRATE IN MILLIMETERS

WW1 - WIDTH OF THE STRIPS IN MILLIMETERS

SS - SEPERATION BETWEEN STRIPS IN MILLIMETERS

FOURTH CARD - FREQGH FORMAT(F10.5)

FREQGH IS THE INITIAL FREQUENCY (GIGAHERTZ)

FIFTH CARD - IN FORMAT(I2)

IN - INDEX NUMBER OF RELATIVE DIELECTRIC PERMITTIVITIES TO BE READ IN NEXT CARDS

SIXTH AND FOLLOWING CARDS CONTAIN VALUES OF EPR2 IN REGION TWO ONE FOR EACH CARD (F10.5)

THE METHOD USED IN FINDING THE EFFECTIVE WAVELENGTH IN THE STRUCTURE IS BASED ON A RCOT SEARCHING ALGORITHM SPECIFICALLY DEVELOPED FOR THIS STUDY.

PROGRAM DEVELOPED BY LT(JG) AHMET M. TUFEKCIOGLU

TURKISH NAVY

THESIS ADVISOR - PROF. JEFFREY B. KNORR PH.D.

NAVAL POSTGRADUATE SCHOOL MONTEREY CALIFORNIA

SEPTEMBER 1974

ALL VARIABLES ARE DOUBLE PRECISION REAL\*8

IMPLICIT REAL\*8(A-H,K-Z)

DIMENSION FYA(80), FYB(80), FYC(80), FYD(80), TRRA(2)

DIMENSION YX(80), YY(80), YZ(80), YC(80), YF(80)

DIMENSION TS1(2), TS2(2), TS3(2), TST(4)

COMMON /FER/ PNS, IFSW

COMMON /ZEYNEP/ S, IEO

COMMON /ACA/ D, WOV, WW

COMMON /AT/ TA, TB, TC, TD, PI, BETAD, TE2, EPR2, IFLG

COMMON /EMRE/ TE, TF, FA2, FA3, FB1, FB2, FB9, FC1, FC2, FD1

EXTERNAL GZR, GZIM

DATA TS1/'DIELECTR', ' FERRITE'/

DATA TS2/' SINGLE', ' COUPLED'/

DATA TS3/' EVEN ', ' ODD '/

READ (5,18) IDF, ISC, IEO

READ (5,19) EPRI, MR1, MR2

READ (5,19) DD, WW1, SS

READ (5,20) FREQGH

READ (5,21) IN



```

PI = 3.14159265358979D0
PISQ = PI**2
EPS = 0.1**1
D = DD*1.E-3
S = SS*1.E-3
Ww = WW1*1.E-3
WCVD = WW/D
FREQDD = FREQGH
C SETTING LOWER AND UPPER LIMITS OF INTEGRATION
BCB = -2.*PI/WW
C BBB = -BCB

DO 17 IK=1,IN
READ (5,20) EPR2
KAK = DSQRT(EPR2)
FREQGH = FREQDD
WRITE (6,22) TS2(ISC),TS1(IDF)
WRITE (6,25) WOVD
IF (ISC.EQ.1) GO TO 1
WRITE (6,26) WW1,SS,TS3(IEO)
GO TO 2
1 WRITE (6,23) WW1
2 WRITE (6,24) EPR2,MR2,DD
IFRS = IDINT(.15/(KAK*D))
C MAXIMUM FREQUENCY RANGE TO BE COVERED
C IN GIGAHERTZ TO AVOID HIGHER ORDER MODES
C IF (IFRS.GT.50) IFRS=50

DO 14 IS=1,IFRS
C IFLG IS A FLAG WHEN EQUALS TO 1 CHARACTERISTIC
C IMPEDANCE CALCULATED. IFSW IS THE FERRITE SWITCH
C WHEN IS EQUAL TO 1 FERRITE SUBSTRATE RELATED
C CALCULATIONS ARE DONE
IFLG = 0
IFSW = 0
FREQ = FREQGH*1.E9
LAM = 3.E8/FREQ

C DOVL = D/LAM
C RCOT FINDING ROUTINE STARTS
C USTL=UPPER LIMIT FOR L/LPR
C ALTL=LOWER LIMIT FOR L/LPR
USTL = KAK-.01
IF (IS-2) 3,4,4
3 ALTL = (USTL+1.)/2.
GO TO 5
4 ALTL = LOVLPR
5 LOVLPR = USTL
ALTL1 = ALTL
IA = 1
C INTERMEDIATE ALGEBRAIC STEPS
6 BETAD = 2.0*PI*DOVL*LOVLPR
TA1 = 240.0*PISQ*DOVL
TA = TA1*MR2
TB1 = DOVL/60.0
TB = TB1*EPR2
TC = TA1*MR1
TD = TB1*EPR1

C DCVLSQ = DOVL**2
TE1 = LOVLPR**2
TE2 = 4.0*PISQ*DOVLSQ
TE = -TE2*(TE1-1.0)
TF = TE2*(EPR2-TE1)
FA2 = TE/TC
FA3 = TF/TA
FB1 = BETAD/TC
FB2 = BETAD/TA
FB9 = TE/TF
FC1 = BETAD/TE
FC1 = TD/TE
FC2 = TB/TF

```



```

C      VALUES OF ALFA WHERE GAMMA 2 SWITCHES
C      REAL TO IMAGINARY
      ALF = DSQRT(TF/D**2)
      ALFN = -ALF
C      EVALUATION OF INTEGRAL
      CALL DQG24 (BCB,ALFN,GZR,Y1)
      CALL DQG24 (ALFN,ALF,GZIM,Y2)
      CALL DQG24 (ALF,BBB,GZR,Y3)
      GZ = Y1+Y2+Y3
      IF (DABS(GZ).LT.EPS) GO TO 11
      IF (IA.NE.1) GO TO 7
      IA = 0
      LCVLPR = ALTL
      GZ1 = GZ
      GC TO 6
7     IF (DABS(GZ1-GZ).GT.DABS(GZ1)) GO TO 9
      IF (ALTL.NE.ALTL1) GO TO 8
      ALTL = ALTL+0.001
      GC TO 10
8     USTL = ALTL
      ALTL = (ALTL1+USTL)/2.
      LCVLPR = ALTL
      GZ1 = GZ
      GC TO 6
9     ALTL1 = ALTL
10    ALTL = (USTL+ALTL)/2.
      LCVLPR = ALTL
      GO TO 6
11    YX(IS) = DOVL
      YY(IS) = 1./LOVLPR
      YZ(IS) = LCVLPR
      YF(IS) = FREQGH
      IFLG = 1
C      EVALUATION OF INTEGRAL FOR CHARACTERISTIC IMPEDANCE.
      CALL DQG24 (BCB,ALFN,GZR,C1)
      CALL DQG24 (ALFN,ALF,GZIM,C2)
      CALL DQG24 (ALF,BBB,GZR,C3)
      CHIMP = (C1+C2+C3)*(1./(8.*PI))*D*2.0
      IF (ISC.EQ.2) CHIMP=CHIMP/2.DO
      YC(IS) = CHIMP
      IF (IDF.NE.2) GO TO 13
C      FERRITE CALCULATIONS STARTS.
C      SET OF PARAMETERS USED FOR SPECIFIC CASE.
      HZERO = 6.000
      WZERO = 2.800*HZERO
      WZERSQ = WZERO**2
      WALFA = 0.10500
      WALFSQ = WALFA**2
      WM = 3.3600
      WMSQ = WM**2
      FERWSQ = (2.*PI*FREQGH)**2
      FERDEN = (WZERSQ-FERWSQ-WALFSQ)**2+4.*WZERSQ*WALFSQ
      XKIPRI = (WM*WZERO*(WZERSQ-FERWSQ+WALFSQ))/FERDEN
      XKIDPR = (WM*WALFA*(WZERSQ+FERWSQ+WALFSQ))/FERDEN
      IFSW = 1
C      INTEGRATION LIMITS DOUBLED FOR FERRITE PART.
      BAB = 2.*BCB
      BUB = -BAB
C      PNS IS POSITIVE NEGATIVE SWITCH FOR +Z AND -Z
C      DIRECTION TRAVELING FIELDS.
      PNS = -1.000
C
C      DO 12 IXX=1,2
C      EVALUATION OF INTEGRAL
      CALL DQG24 (BAB,ALFN,GZR,FER1)
      CALL DQG24 (ALFN,ALF,GZIM,FER2)
      CALL DQG24 (ALF,BUB,GZR,FER3)
      TRRA(IXX) = FER1+FER2+FER3
      PNS = PNS**2
12    CONTINUE
C
      MRMO = 4.D-7*PI

```



```

SABIT = (FREQ*D*MRMO)/(4.*CHIMP)
FYA(IS) = DABS(XKIPRI*TRRA(1)*SABIT)
FYB(IS) = DABS(XKIPRI*TRRA(2)*SABIT)
FYC(IS) = DABS(XKIDPR*TRRA(1)*SABIT)
FYD(IS) = DABS(XKIDPR*TRRA(2)*SABIT)
BETTA = BETAD/D
FYA(IS) = (BETTA+FYA(IS))/BETTA
FYB(IS) = (BETTA+FYB(IS))/BETTA
FYC(IS) = 8.686*2.*PI*FYC(IS)/(BETTA*FYA(IS))
FYD(IS) = 8.686*2.*PI*FYD(IS)/(BETTA*FYA(IS))
13 FREQGH = FREQGH+1.000
14 CONTINUE
C
WRITE (6,27)
WRITE (6,28)
C
DC 15 ITR=1,IFRS
15 WRITE (6,29) YF(ITR),YX(ITR),YZ(ITR),YY(ITR),YC(ITR)
C
IF (IDF.NE.2) GO TO 17
WRITE (6,30)
C
DC 16 IH=1,IFRS
16 WRITE (6,31) YF(IH),FYA(IH),FYB(IH),FYC(IH),FYD(IH)
C
17 CONTINUE
C
STOP
C
18 FCRMAT (3I11)
19 FORMAT (3F10.5)
20 FORMAT (F10.5)
21 FORMAT (I2)
22 FCRMAT ('1',//55X,'MICROSTRIP ANALYSIS'//55X,A8,2X,
1 'STRIP'//55X,A8,2X,'SUBSTRATE'//)
23 FCRMAT ('0',55X,'WIDTH OF STRIP=',F5.2)
24 FCRMAT ('0',55X,'EPR2=',F5.2,2X,'MR2=',F5.2//56X,
1 'THICKNESS D=',F4.1,2X,'MM'//)
25 FCRMAT ('0',60X,13(1H*)/2(61X,1H*,11X,1H*//),61X,1H*,
1 'W/D=',F4.1,3X,
2 1H*/2(61X,1H*,11X,1H*//),61X,13(1H*)//)
26 FCRMAT ('0',55X,'WIDTH OF STRIP=',F4.1,2X,'MM'//
1 56X,'SEPERATION',F4.1,2X,'MM'//55X,A8,2X,'MODE')
27 FCRMAT ('0',32X,'FREQUENCY ',2X,' D/LAMBDA ',5X,
1 'L/LPR',6X,'LPR/L',5X,'CH IMPEDANCE')
28 FCRMAT (' ',30X,5(2X,10(1H_))//)
29 FCRMAT ('0',32X,F6.3,1X,'GHZ',3(2X,F10.5),2X,
1 F6.2,1X,'OHM'//)
30 FCRMAT ('0',8X,'FREQUENCY',8X,'BF-/BPR ',10X,
1 'BF+/BPR',10X,'2*PI*8.686*DELALFA/B **NEG AND POS')
31 FCRMAT ('0',3X,F10.3,4(4X,F15.6))
END

```



C

```

REAL FUNCTION GZR*8(ALFA)
ROUTINE DEFINES THE POLYNOMIAL FOR HYPERBOLIC CASE
IMPLICIT REAL*8(A-H,K-Z)
COMMON /FER/ PNS,IFSW
COMMON /ACA/ D,WOVD,WW
COMMON /AT/ TA,TB,TC,TD,PI,BETAD,TE2,EPR2,IFLG
COMMON /EMRE/ TE,TF,FA2,FA3,FB1,FB2,FB9,FC1,FC2,FD1
ALFAD = ALFA*D
TG1 = ALFAD**2
TG = TG1-TE
VG = DSQRT(TG)
BB = FB1*ALFAD/VG
BC = FC1*ALFAD
TH = TG1-TF
VH = DSQRT(TH)
FA1 = 1./DTANH(VH)
BA = FA1*FB2*ALFAD/VH
FFF1 = -(FA2/VG+FA3*FA1/VH)
FFF2 = BB+BA
FFF3 = -FFF2
FFF4 = BC*(BB+BA*FB9)-(FC1*VG+FC2*FA1*VH)
DENOM = FFF1*FFF4-FFF2*FFF3
GG4 = -FFF1/DENOM
GG2 = FFF2/DENOM
CALL TRAN (ALFA,ALFAD,GX)
IF (IFLG.EQ.1) GO TO 1
GZR = GG4*GX
RETURN
1
RCAA = GG4/TE
RCAA1 = ALFAD*FD1*GG4
RCAB = (1./(TC*VG))*(GG2-RCAA1)
RRA = GX*RCAA**2
RRB = GX*RCAB**2
RCA = RCAA*RCAB*GX
RCAC = GG4/(TF*DSINH(VH))
RCAD = 1./(TA*VH*DSINH(VH))
RCAE = -GG2+RCAA1*FB9
RCB = RCAC*RCAD*RCAE*GX
RRC = GX*RCAC**2
RRD = GX*(RCAD*RCAE)**2
P1B = ((TG1+TG)/VG)*(TD*RRA+TC*RRB)*BETAD
P1C = 2.*ALFAD*(BETAD**2+TE2)*RCA
P1INT = P1B+P1C
RKA1 = DSINH(2.*VH)
RKA = RKA1-(2.*VH)
RKB = TG1*BETAD*TB*RRC/VH
RKC = BETAD*TA*VH*RRD
TAVTB = TE2*EPR2
RKD = ALFAD*((TAVTB)+BETAD**2)*RCB
RKE = RKB+RKC-RKD
RKG = RKA1+(2.*VH)
RKH = TG1*BETAD*TA*RRD/VH
RKI = BETAD*TB*VH*RRC
RKK = (TAVTB+BETAD**2)*ALFAD*RCB
P2INT = RKA*RKE+RKG*(RKH+RKI-RKK)
IF (IFSW.EQ.1) GO TO 2
GZR = P1INT+P2INT*0.5
RETURN
2
FERAR = RKA/(2.*VH)
FERB = (-TG1*BETAD**2+TF**2)*RRD
FERC = PNS*TB*BETAD*ALFAD*VH**2.0*RCB
FERD = TH*RRC*TB**2
GZR = FERAR*(FERB+FERC-FERD)
RETURN
END

```



```

REAL FUNCTION GZIM*8(ALFA)
IMPLICIT REAL*8(A-H,K-Z)
COMMON /FER/ PNS,IFSW
COMMON /AT/ TA,TB,TC,TD,PI,BETAD,TE2,EPR2,IFLG
COMMON /ADA/ D,WCDV,WW
COMMON /EMRE/ TE,TF,FA2,FA3,FB1,FB2,FB9,FC1,FC2,FD1
ALFAD = ALFA*D
TG1 = ALFAD**2
TG = TG1-TE
VG = DSQRT(TG)
BB = FB1*ALFAD/VG
BC = FD1*ALFAD
TH = -TG1+TF
VH = DSQRT(TH)
FA1 = DCOTAN(VH)
BA = FA1*FB2*ALFAD/VH
FFF1 = -(FA2/VG-FA3*FA1/VH)
FFF2 = BB-BA
FFF3 = -FFF2
FFF4 = BC*(BB-BA*FB9)-(FC1*VG+FC2*FA1*VH)
DENOM = FFF1*FFF4-FFF2*FFF3
GG4 = -FFF1/DENOM
GG2 = FFF2/DENOM
CALL TRAN (ALFA,ALFAD,GX)
IF (IFLG.EQ.1) GO TO 1
GZIM = GG4*GX
RETURN
1 RCAA = GG4/TE
RCAA1 = ALFAD*FD1*GG4
RCAB = (1./(TC*VG))*(GG2-RCAA1)
RRA = GX*RCAA**2
RRB = GX*RCAB**2
RCA = RCAA*RCAB*GX
RCAC = GG4/(TF*DSIN(VH))
RCAD = 1./(TA*VH*DSIN(VH))
RCAE = GG2-RCAA1*FB9
RCB = RCAC*RCAD*RCAE*GX
RRC = GX*RCAC**2
RRD = GX*(RCAD*RCAE)**2
PIB = ((TG1+TG)/VG)*(TD*RRA+TC*RRB)*BETAD
PIC = 2.*ALFAD*(BETAD**2+TE2)*RCA
PIINT = PIB+PIC
RKA1 = DSIN(2.*VH)
RKA = 2.*VH-RKA1
RKB = TG1*BETAD*TB*RRC/VH
RKC = BETAD*TA*VH*RRD
TAVTB = TE2*EPR2
RKD = ALFAD*((TAVTB)+BETAD**2)*RCB
RKE = RKB+RKC+RKD
RKG = RKA1+(2.*VH)
RKH = TG1*BETAD*TA*RRD/VH
RKI = BETAD*TB*VH*RRC
RKK = (TAVTB+BETAD**2)*ALFAD*RCB
P2INT = RKA*RKE+RKG*(RKH+RKI-RKK)
IF (IFSW.EQ.1) GO TO 2
GZIM = PIINT+P2INT*0.5
RETURN
2 FERAI = (RKA1+2.*VH)/(2.*VH)
FERB = (-TG1*BETAD**2+TF**2)*RRD
FERC = PNS*TB*BETAD*ALFAD*VH*2.0*RCB
FERD = TH*RRC*TB**2
GZIM = FERAI*(FERB+FERC-FERD)
RETURN
END

```



```

SUBROUTINE TRAN (ALTR,ALFD,GXTR)
IMPLICIT REAL*8(A-H,K-Z)
COMMON /ZEYNEP/ S,IEO
COMMON /ADA/ D,WQVD,WW
CB = ALFD*WQVD*0.5
GX1 = ((1./CB)*(DSIN(CB)))**2
IF (IEO.EQ.0) GO TO 2
CD = ALTR*(S+WW)*0.5
IF (IEO.EQ.2) GO TO 1
GX1 = GX1*(2.*DCOS(CD))**2
GO TO 2
1 GX1 = GX1*(2.*DSIN(CD))**2
2 GXTR = GX1
RETURN
END

```



## LIST OF REFERENCES

1. R. E. Collin, Field Theory of Guided Waves, pp. 179-180, McGraw-Hill Book Co., New York, 1960.
2. E. Denlinger, "A Frequency Dependent Solution for Microstrip Transmission Lines," IEEE Trans Microwave Theory and Techniques Vol. MTT-19, pp.30-39, January 1971.
3. A. Farrar and A. T. Adams, "Characteristic Impedance of Microstrip by the Method of Moments," IEEE Trans Microwave Theory and Techniques Vol. MTT-7, January 1970, pp. 65-66.
4. R. Harrington, "Matrix Methods for Field Problems," IEEE Proceedings, Vol. 55, pp. 136-138, February 1967.
5. J. Helszajn, Principles of Microwave Engineering, pp. 53-65 and pp. 117-130, Wiley, 1969.
6. T. Itoh and R. Mittra, "Spectral-Domain Approach for Calculating the Dispersion Characteristics of Microstrip Lines," IEEE Trans Microwave Theory and Techniques, Vol. MMT-21, pp. 496-499, July 1973.
7. J. B. Knorr, "Perturbation of Wave Guide Material," Unpublished Class Notes, Naval Postgraduate School, Monterey, California.
8. J. B. Knorr, "Analysis of Single and Coupled Microstrip," Unpublished Class Notes, Naval Postgraduate School, Monterey, California.
9. M. Krage and G. Haddad, "Frequency Dependent Characteristic of Microstrip Transmission Lines," IEEE Trans Microwave Theory and Techniques, Vol. MTT-20, pp. 678-688, October 1972.
10. Microwave Engineering Handbook Vol 1., p. 132-137, Artech House Inc., 1971.
11. J. Weiss, et. al., "Parameters of Microstrip Transmission Lines and of Coupled Pairs of Microstrip Lines," IEEE Trans Microwave Theory and Techniques, Vol. MTT-16, pp. 1021-1026, December 1968.



# INITIAL DISTRIBUTION LIST

	No. Copies
1. Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2. Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3. Professor Richard W. Adler, Code 52Ab Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
4. Professor Jeffery B. Knorr, Code 52Ko Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	2
5. LT (jg) Ahmet M. Tüfekçioğlu, Turkish Navy Başa Sok.Fırtına Apt. No. 13/21 Levent-Istanbul TURKEY	2
6. Istanbul Teknik Üniversitesi Elektrik Fakültesi Taskısla, Istanbul, TURKEY	1
7. Orta Doğu Teknik Üniversitesi Elektrik-Elektronik Fakültesi Ankara, TURKEY	1
8. Professor Sydney R. Parker, Code 52 Department of Electrical Engineering Naval Postgraduate School Monterey, California 93940	1
9. Boğaziçi Teknik Üniversitesi Bebek, Istanbul, TURKEY	1







15 AUG 75

20323  
23593

Thesis

156138

T85

Tüfekcioğlu

c.1

Hybrid mode analysis of  
microstrip on dielectric  
and ferrite substrate.

15 AUG 75

20323  
23593

156138

Thesis

T85

Tüfekcioğlu

c.1

Hybrid mode analysis of  
microstrip on dielectric  
and ferrite substrate.

thesT85

Hybrid mode analysis of microstrip on di



3 2768 000 98393 6

DUDLEY KNOX LIBRARY